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ABSTRACT

The current status of mathematics education in the Union of Soviet Socialist Republics is reported. The perceptions of Davis and Romberg reflect their own observations and discussions with Soviet educators. Kantowski and Rachlin give their impressions resulting from reading reports on Soviet research techniques and findings. Davis and Romberg discuss common practices in present Soviet schools, difficulties in language, Soviet mathematics curricula, and mathematics education research and development in the Soviet Union. Rachlin considers Soviet approaches to the study of problem-solving processes in mathematics, while Kantowski gives another view of the value of studying mathematics education research and development in the Soviet Union. (MK)

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# AN ANALYSIS OF MATHEMATICS EDUCATION IN THE UNION OF SOVIET SOCIALIST REPUBLICS

Robert B. Davis
Thomas A. Romberg
Sidney Rachlin
Mary G. Kantowski

ERIC Clearinghouse for Science, Mathematics
and Environmental Education
The Ohio State University
College of Education
1200 Chambers Road, Third Floor
Columbus, Ohio 43212

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# Mathematics Education Reports

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## Editor's Foreword

This publication was prepared at the invitation of the National Institute of Education to report on the current status of mathematics education in the Union of Soviet Socialist Republics. It was developed by two persons who visited the Soviet Union a year ago -- Bob Davis -- and Tom Romberg -- and two persons who have spent the past several years delving deeply into the available Soviet literature base in mathematics education -- Mary Grace Kantowski and Sid Rachlin. They were aided by the specific knowledge of others, whom they acknowledge.

The chapters unfold the perceptions of Davis and Romberg, writing from the immedicacy of their visit with its kaleidoscope of observations of Soviet schools and discussions with leading Soviet educators. In contrast are the careful analyses of Kantowski and Rachlin, unfolding their impressions gleaned from reading reports on Soviet research techniques and findings. The curriculum in Soviet schools, instructional practices, research ideas, and the general milieu of the students, teachers, parents, and others in a system that has different heritage, and thus different strengths and weaknesses, from the American system are presented.

ERIC/SMEAC is pleased to make this book available to readers. It provides information and impressions, and indicates some benefits of international comparisons. The need for a reanalysis of what research and development in curriculum and instruction are all about is thus apparent.

Marilyn N. Suydam
Editor

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*Table of Contents*

# Chapter I

## Explanation of the Present Document

Robert B. Davis

In the spring of 1979, the National Institute of Education requested the preparation of an analytic report on "Mathematics Education in the Union of Soviet Socialist Republics" that should include information on the following matters:

1) A description of the state of educational research, development; and practice in the U.S.S.R. in the area of the teaching and learning of mathematics.

2) Opportunities for the United States to conduct observational research on Soviet practices in the teaching and learning of mathematics, and of research and development in mathematics education.

3) Areas in which Soviet research could enhance U.S. researchers' ability to define research questions or approach research problems differently, or in which Soviet practice might be especially relevant to the U.S.

4) A selected bibliography of the most important publications in the field.

N.I.E. further requested that particularly careful attention should be paid to two matters: the effective improvement of actual daily classroom practice, and the question of educational equity.

The present report is a response to this assignment, prepared by a group of U.S. researchers each of whom has some knowledge of Soviet work.

The form of this report deserves explanation. Any analysis of Soviet research and development ("R and D") in mathematics education involves some fundamental judgments that are matters of disagreement even among reasonable people. At first glance, one might say that research seeks the truth about reality, and development is concerned both with the improvement of practice and with the creation of better tools for future research. Isn't this clearly objective, and raised above the level of possible disagreements?

In fact, it is not. Whatever R and D is done grows out of the reality of existing classroom practices—which vary considerably, and are not the same in the U.S. and in the U.S.S.R.—and is done in response to perceptions of what is needed, what is possible, and what would be valuable. But there are variations of a more subtle sort.

Research grows out of existing conceptualizations. As Kuhn (1962) points out, when Lavoisier and Priestley independently heated an orange powder and obtained a colorless gas, their realities were essentially the same. Their conceptualizations, however, were entirely different. Priestley conceptualized this reality in terms of adding the fluid caloric to the powder, and obtaining dephlogisticated air as a result. Lavoisier, on the other hand, invented the idea of "elements" and "compounds," and concluded that the red powder was a compound which, upon being heated, had decomposed into its elements. Despite correspondence between them, neither was able to convince the other.

It may well be the case that a human being "thinking mathematical thoughts," and human students in a social situation which is intended to help them learn mathematics, are both realities at least as complex as the orange powder of Lavoisier and Priestley, and at least as susceptible to alternative conceptualization. Indeed, anyone familiar with mathematics education over the past five or six decades can hardly doubt it.

*In fact, a major part of the value in studying Soviet mathematics education is to learn from the contrasts between their conceptualizations and those that are current in the United States.* One must realize that the goal is not to decide who is right and who is wrong. Probably no conceptualization represents the whole truth. The goal, rather, is to compare and contrast alternative conceptualizations, thereby moving on to new and improved conceptualizations.

But even within the United States there are different conceptualizations, both as to how to analyze "mathematical thought" in all its diverse ramifications, and also as to how to analyze the social settings and activities that constitute the teaching and learning of mathematics. Mathematical thought is considered by some observers to be the rote process of carrying out an explicit memorized algorithmic procedure, by others to be a sophisticated process of combining procedures and creatively modifying procedures (even to the point of inventing new ones), and by yet other observers to be a complex process involving metalanguage, heuristics, and so forth. Over the content of mathematics to be learned there are disagreements, as there are over the questions of what kind of behaviors to expect, and under what conditions. (Should a calculus test be open-book? Should students be allowed to use hand-held calculators on a test? Should students know immediately the factorization of $x^3 + 1$, or may they take time to work it out? Should mathematics tests be timed, or should students be allowed all the time they need? Should two students be allowed to collaborate in solving a problem? Must students be able to justify on logical grounds the steps they take? The list of uncertainties is very long, and range from minute details to the most sweeping fundamentals.)

We are, in general, more interested in major differences in conceptualizations than in details. If the phenomena themselves are viewed differently, there are perhaps even greater differences in answers to the question of how to study these phenomena and what sort of "knowledge" to seek. Are case studies useful? Should individual differences

be described in terms of a small number of variables, mostly numerical (as in reporting that a student has an I.Q. of 142)? Should one observe primarily (or solely) the products of a student's mathematical effort (which usually means the student's final answers), or should one study the process by which these answers were produced?

Because of the range of different conceptualizations, even among present-day U.S. experts, this report has been prepared by a group of collaborators, each of whom attempts to build on his or her own conceptualizations, values, and judgments. We believe this offers a sounder basis for conclusions than could be obtained from any single perspective. In point of fact, however, there is a fairly substantial agreement on the main matters that are discussed. (The authors of the various chapters are explicitly identified. In addition to the main group of collaborators—Sidney Rachlin, The University of Calgary; Thomas Romberg, The University of Wisconsin-Madison; Mary Grace Kantowski, University of Florida; and Robert B. Davis, University of Illinois, Urbana/Champaign—important assistance has been received from Bruce Vogeli, Jeremy Kilpatrick, Walter Reitman, Izaak Wirszup, Leon Henkin, Lee Shulman, Donald Michie, Peter Maggs, and Harold Hodgkinson.)

Given the uncertainties, ambiguities, lacunae, and disagreements that we have encountered in our attempts to assemble relevant descriptive materials, we are well aware that there are almost certainly both gaps and errors in our final result. We ask readers to accept this report for what it is: the best description of Soviet R and D in mathematics education, and its relevance to U.S. practice, that we could put together within reasonable constraints on time and other resources. Readers who send us corrections or additions will find us grateful, and not disposed to argue.

## Reference

Kuhn, Thomas S. The Structure of Scientific Revolutions. Chicago: University of Chicago Press, 1962.

# Chapter II

## Common Practices in Present Soviet Schools

Robert B. Davis and Thomas A. Romberg[1]

### A. *Some General Remarks*

Within the past 15 months, both of us have had opportunities, as official U.S. delegates and guests of the Soviet government, to visit schools in several cities in the U.S.S.R.[2] Considering the size of the U.S.S.R., the diversity of languages[3] and cultures which it includes (e.g., Eskimoes), the range of life styles (from peasants to nuclear physicists, from sparsely populated rural areas to Moscow and Leningrad, from the Baltic to the Arctic), and the range of cultures (northern European, southern European, Asian, and so forth), we suspect that at best we have seen very, very little of schooling in the U.S.S.R. There is the further problem that we have seen a non-random sample of Soviet schools. Yet both of us independently arrived at similar conclusions, which have subsequently been compared with those of Bronfenbrenner (1970) and of Smith (1977) and found to be substantially similar. Hence, we have some confidence that our perceptions correspond to at least a piece of the reality of Soviet schools.

1. The first thing that strikes you, the moment you enter a Soviet school, is the orderliness. Children are polite to adults, polite to one another, and attentive during lectures. For U.S. visitors, this is a surprising novelty.

2. To the preceding remark another must be added immediately: a visitor can see no evidence of blatant force or compulsion. Architecturally, schools are reasonably open and inviting. There are no police in sight. Windows are not barred, there is no great prevalence of locked doors, keys, and so forth. Clearly, there is adult control —a world of children would be far less orderly—but the controls are subtle, even warm and loving.

3. That brings us to a third point, on which we rely primarily on the reports of Urie Bronfenbrenner, Hedrick Smith, and Harold Hodgkinson: the Russians give a very high priority, and a deep, warm affection, to children. The Soviets provide food, clothing, and

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[1]We wish to acknowledge the valuable assistance of Bruce Vogeli and Harold Hodgkinson in putting together this view of Russian schools.

[2]Specifically, Moscow, Leningrad, Tallin, and Kiev.

[3]For example, it is worth recalling that Russian is the principal language of less than 50 percent of the population of the U.S.S.R.

medical care lavishly to children—far more lavishly than to adults. Children who don't even like caviar may get some, at the expense of adults who crave it (and it is typically in short supply). The Soviets claim that every school has its own full-time physician, despite evidence of inadequate medical care of adults. Grandmothers wait outside schools to walk home with their grandchildren. When the movie *The Russians Are Coming* portrays Soviet sailors abandoning other activities to rescue a child who is in danger, the film is true to a deep trait in Russian character. Many generally similar incidents have been reported. This looks very strange to the eyes of an American visitor.

Workers, astronauts, and top-level scientists all find time to work with young people: the astronaut Yurie Gagarin sponsors a club in the Moscow Pioneer Palace; A. N. Kolmogoroff, one of the greatest mathematicians of the twentieth century, sponsors a high school and *teaches in it three times a week.* (The tradition is not new: Leo Tolstoy created and operated a school for peasant children.)

Responsibility for young children is enthusiastically accepted even by teenagers; there are many reports of teenagers spontaneously (and affectionately) showing care and concern for children, strangers to them, whom they encounter on the street.

4. One of the most striking characteristics of Soviet schools and of Soviet life in general, is the *collective.* A student is part of a group of students, and every student in the group has responsibility for every other student in the group. If a student is not learning satisfactorily, the group—called the "collective"—must deal with the problem. Similarly, if a student is not doing his or her homework, or is being tardy, the group takes action.

Nor is the collective limited to students in school. Even the American visitor, traveling for two weeks with a group of Soviet and American educators, finds that he or she is a member of a collective. In fact, it can be a surprisingly supportive and reassuring environment—one is never alone, never faces alone the problems of currency or transportation or housing. The welfare of every member of the group is the business of—indeed, the responsibility of—every member of the group.

5. Hedrick Smith reports on "parents night" in a Soviet school. In the U.S., parents night is typically an occasion for teachers to report to parents, to be questioned by parents, not infrequently to be accosted by parental complaints, requests, and demands. One might give the capsule description: parents sit in judgment on teachers.

Parents night in the U.S.S.R. is quite the opposite. The voice of authority is the teacher's. If Ivan has not been doing his homework, this deficiency will be publicly proclaimed, and Ivan's parents

will be publicly admonished and told that they must correct the problem.[4] There are even reports of authorities restricting the vacations awarded to parents of children who are doing poorly in school.[5]

6. The general atmosphere in schools is regularly reported as warm, caring, even loving.

7. If a great deal is given to children, that does NOT include individual choice. The same is true for visiting U.S. guests: the collective provides good meals and excellent entertainment, but if this evening's schedule calls for going to the circus, Russians will not be prepared for an individual member of the collective to express a preference for going to the ballet. That sort of thing simply isn't done.

8. In regard to the general appearance of Soviet schools that we visited, physically the rooms were clean, well-lit, well-equipped, and attractively decorated. The walls displayed a profusion of social consciousness statements ("A Pioneer does his homework carefully." "A Pioneer obeys his parents."). Classrooms had paired rows of desks firmly fixed to the floor. Students were dressed in uniforms, raised their hands to be called on, and stood up to recite. However, students did talk quietly to their deskmates, with whom they share answers and discuss the work. [Indeed, Soviet students spend more time copying answers from other students, or from the teacher's work, than an American visitor is prepared to accept. This copying is NOT cheating: it is an official (and very common) form of "helping."]

9. As to the role of Soviet teachers, beginning with the fourth grade, mathematics is *always* taught by a specialist teacher *who teaches nothing but mathematics*. This applies to every grade level, from grade four upward.

Furthermore, it is always taught in the same way in every classroom throughout the country. Teachers follow a prescribed didactic sequence involving first a lecture, then questions. Students follow in somewhat rote fashion, upon the prodding of teachers. In the Soviet instructional system, the teacher is viewed as a conduit. Teachers are NOT supposed to adapt or change materials to meet the needs of their students. Teachers are not seen as professional decision-makers responsible for planning the curriculum.

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[4] Here again we see *the collective* in operation. "Home assignments"—work that a child should do at home—is, in effect, a responsibility of the "home collective." The family is responsible for making sure that Ivan gets the work done, and assists him with it. If the child does not get it done, it is not his fault alone: it is the whole family's fault.

[5] Parents are expected to assist the school not only in the matter of home assignments, but in other things as well. We have visited schools and found parents making bookshelves, flooding a playground to make an ice rink, etc. Parents are expected to assist in the operation of the school.

The job of a school teacher in the U.S.S.R. is to carry out a well-designed, carefully elaborated program of instruction. Indeed, inspectors check on teachers to see that they are teaching the syllabus as intended. If a teacher wants to adapt a program of instruction, it is expected that he or she will get permission and an inspector will in turn check to see that there is a rationale for making the adaptation, and that the results of the adaptation are appropriate. It is assumed that teachers treat all students equally.

In summary, teaching in a Soviet classroom is more formal and institutionally structured than in U.S. classrooms. The teacher's role is functionally prescribed. Nonetheless—perhaps surprisingly—the school atmosphere is markedly one that is caring and supportive.

10. The organization of the schools differs from that in the U.S. The education of the young is defined differently in the U.S.S.R., as we shall see in later sections. But it would be fair to say that, to begin with, it is split into two parts: schools and Pioneer Palaces.

From morning through early afternoon each child attends a school. In late afternoon, nearly all children attend a quite different institution, known as a Pioneer Palace. The Pioneer Palace—possibly one of the most impressive structures in town—is something like a combination, in a single institution and a single building, of what in the U.S. would be parceled out to many different institutions: the YMCA/YWCA, museums, zoos, Boy Scouts and Girl Scouts, piano lessons, music clubs, student orchestras, computer clubs, ballet lessons, and so forth.

In a Pioneer Palace you may find:

a) a "Friendship Club," composed of children who are learning English, and who correspond with children in school in various English-speaking countries around the world, read books (in English) from the U.S., etc.

b) ballet classes (Russia is, after all, Russia)

c) a computer club, where students make and use computers

d) a "future astronauts" club, which has flight simulators and other equipment

e) a radio club, where students make radios

f) a zoo, where students care for the animals

g) a "future farmers" club, where students learn about agriculture

h) classes in painting, ceramics, and sculpture

i) cooking classes

j) classes in dressmaking

k) classes in interior decoration

l) science clubs

m) swimming instruction

n) classes in how to set a table, how to entertain, "good manners," etc.

o) a model railroad club

p) an aquarium cared for by students

q) a drama club

and special interest clubs of other types.

The personnel at a Pioneer Palace have quite different educational backgrounds from those in schools. They may be chemists, working part time (perhaps as volunteers) at the Pioneer Palace. Some actually are professional astronauts.

One immediate consequence of the dual system is important for U.S. educators: because the "activity" and "special interest" classes are in the Pioneer Palace, they are *not* in schools. The school is a formal, academic place. The familiar U.S. problem of seeing demanding courses in Euclidean geometry or English losing out in the competition with more "exciting" (and less demanding) courses in film-making, shop, the lyrics of rock music, or television viewing cannot occur in Soviet schools. These "activity" and "special interest" courses *are not part of the school at all*. They happen at a different time and in a different place. The school deals with the serious academic subjects.

## B. *Soviet Schools and Curricula*

Deferring, for the moment, any further discussion of Pioneer Palaces, we turn exclusively to the question of Soviet schools, which children typically attend for the morning and the earlier part of the afternoon. Descriptions of the Soviet system vary somewhat (as do descriptions of the U.S. system), but the general pattern seems to be as follows.

C.

### *General Pattern*

1. For the youngest children—below age 7—a new system of early childhood education has recently been devised, and is now in the process of being implemented. These schools are an impressive innovation, accepting (if parents request) even very young infants, and placing equal emphasis on academic learning, "up-bringing" (an interesting

Soviet feature which we discuss below), physical care, and social development. These schools are unmistakably worthy of study in themselves; since, however, they do relatively little in the way of teaching mathematics, we do not consider them extensively in the present report.

2. For children from age 7 to age 15 there is a single kind of school, sometimes called the "eight-year school." This may, in fact, be a separate school, or may be part of a so-called "ten-year school," enrolling students from age 7 through age 17 (Shabanowitz, 1978, p. 25).

3. The first three years of the "eight-year school" (or of the "ten-year school") may be separated, constituting a primary school (Shabanowitz, 1978, p. 25), although just how distinct these primary schools are (or, conversely, how completely integrated into the eight- or ten-year school) is a matter on which various reports do not agree. The distinction may or may not be important—within the U.S., for example, there are isolated cases where all fourth grades meet in a separate building, cases where grades 1-3 meet in one building, and grades 4-6 meet in a separate building, cases where fifth grades meet in a separate building, and so on. These special arrangements are usually either the result of attempts to make effective use of existing buildings of varying sizes, or are caused by constraints in school bus schedules, or represent attempts at improving racial integration. Perhaps they do not alter the broad pattern of U.S. schools, but considering them can greatly—and perhaps disproportionately—complicate the task of *describing* U.S. schools. Presumably there are similar variations among Soviet schools.

4. Thus the eight- or ten-year school can be thought of, at least roughly, as a Soviet equivalent of grades 1 through 12 in the United States. What is especially striking to U.S. observers is the absence of "tracking" or "homogenized classes" in the usual eight- or ten-year schools. The Soviets believe strongly in the goal of giving all citizens the same education—at least in this basic sense—and are very proud of their success in achieving this. To American eyes this is very strange indeed, perhaps even incredible. Within the scope of these eight- or ten-year schools there is apparently no division into a "college preparatory" track, a "business" or "retail" track, a "vocational" or "shop" track, or other tracks. There is just one single version of education, the same for everybody.

Well, not quite everybody, as we shall presently see. The Soviets have some remarkable arrangements for a small number of gifted students, and provide some other forms of variation. But for most students, from the time they are 7 until they are 15 or so, there is a single basic educational program, exactly the same for everyone.

5. Attached to the top end of the eight-year school (thereby transforming it to a "ten-year school"), or perhaps existing separately, one finds at least four kinds of schools: the "secondary general polytechnical school" for ages 15 to 17, the "secondary specialized schools" for ages 15 to 19, the "evening shift" schools and the "correspondence

schools" for ages 15 to 18, and the "vocational-technical schools" (Shabanowitz, 1978, pp. 24-25). It is at this point that the major alternatives in Soviet education—analogous to "tracking" in the U.S. —make their appearance. These alternatives correspond roughly to U.S. alternatives provided by "vocational" tracks, by "commercial" tracks, by "college preparatory" tracks, and by community colleges, although we shall not try to match the possible route of a Soviet student through the various schools with corresponding routes of U.S. students along the variety of paths that are possible here. Our present data are not good enough to justify such an attempt at making correspondences.

6. Soviet schools meet six days a week, Monday through Saturday. The school year lasts from 35 weeks to 38 weeks, depending upon grade level.

7. According to Shabanowitz (1978, p. 25) a student who receives unsatisfactory grades in three or more subjects must repeat that year (in U.S. language, the student would be "kept back" or "not promoted").

8. Shabanowitz (1978, p. 26) also reports that any Soviet citizen who completes (at least) grade 10 is eligible for admission to higher education. There are, however, competitive examinations for admission to higher education programs.

9. The basic eight-year school is reported as offering the following subjects: Russian language, world literature, world history, social sciences, natural history, geography, biology, physics, astronomy, drawing, chemistry, English, other foreign languages, physical culture, labor training—and, of course, mathematics. *This is reported as the universal curriculum for all Soviet students from age 7 to 15.*

(One has the very strong sense that something here is being mistranslated, or otherwise misunderstood; a persistent theme in U.S. attempts to describe and understand Soviet educational practice—and even more so in the case of research and educational theory—is the need for clarification and specificity. There is no shortage of reasons for doubting the completeness and correctness of our present descriptions. Getting clear and accurate descriptions could be a strong first step toward more effective communication. Certainly, a school program as heavily academic as the one just described, required for *every* student, would not be considered a realistic possibility in the United States at the present time. How, then, can it succeed in the Soviet Union? There is much here that requires further clarification.)

10. The present situation can, to some extent, be clarified by considering how Russian and Soviet education has evolved. The history of Russian schooling can be seen as the story of trying to accomplish four main goals:

a) taking the diverse collection of schools of the Russian past—ranging from the Smolny Institute "for young ladies of noble birth," founded in 1764, and the Moscow School of Mathematical

and Navigational Sciences, founded by Peter the Great in 1701 (which, at that time, was one of the leading mathematical schools in the world), to one-year village schools that taught some reading, some writing, and the four operations of addition, subtraction, multiplication, and division—and from such beginnings creating a unified nation-wide system of schools that would provide identical educational opportunities to every child in the U.S.S.R.;

b) within the framework of a deep and publicly declared belief in the equality of all humans, and in the desirability of an egalitarian society, somehow providing for individual differences in people, and for the national economic need for a diversified work force;

c) relating education to the practical world of work, including the provision of "labor experiences" for all students;

d) using systematic study ("educational research") as a tool to improve the operation of the educational system.

A great deal of what has happened can be seen in relation to these four themes. In particular, there has been a history of efforts to get all children to work in some industry or productive labor, countered by an opposing pressure to keep students at work learning trigonometry and calculus and English instead of "running off to the countryside to help bring in the crops." The more-work-vs.-more-study pendulum has already had several swings (cf., e.g., Vogeli, 1968; Vogeli, 1971; Shabanowitz, 1978). Prominent Soviet professional mathematicians have argued the need to modernize the mathematical content taught in schools, which is perhaps one aspect of the general problem of relating school to the outside-of-school world of work—in this case, to professional work in mathematics, science, and engineering. In most cases, however, "labor education" probably refers more to work in factories and on farms than it does to sophisticated professions.

11. "Kindergarten" in the U.S.S.R. is sometimes taken to mean a school that is attended for as long as four years, from age three until age seven. The actual patterns of attendance seem to vary considerably; for example, Taruntaeva (1971) says: "If the children previously attended nursery school, and directly entered the second of the four kindergarten years from their first year, mathematics activities can begin...immediately after September." The interesting word here, of course, is "If."

The mathematical content dealt with in the four years of kindergarten is described as: "numbers and counting," "size," "shape," "orientation and space," and "orientation and time." This work may include the beginnings of a fairly wide range of mathematical skills and understandings. Taruntaeva (1971) lists "vision of a whole object as it can be divided into equal parts" and "development of the visual estimation ability of the children," as well as "orientation in space—to be able to determine the position of some object not only with

respect to [the student] himself, but also with respect to some other person." Vocabulary includes: cube, large, small, short, very short, square, triangle, left side, right side, morning, day, evening, night, widest, narrower, circle, rectangle, sphere, cylinder, in front of me, above me, today, tomorrow, fast, slow, and so on.

12. As a further attempt to define and clarify Soviet practice, consider the third-grade curriculum. The "third grade" in an eight-year or ten-year school enrolls nine- and ten-year-olds. [Notice, as this instance suggests, that "grade 3" in Soviet schools is something of a match, in terms of the chronological age of the students, to "grade 5" (or at least "grade 4") in the U.S. With this in mind, much of the Soviet mathematics curriculum does not seem unduly advanced, but some isolated items do seem surprisingly advanced (e.g., proofs by mathematical induction in grade 9).] Pchelko (1971) reports that the mathematical content of grade 3 includes:

skill in reading and writing multi-digit numbers

the place-value meaning of digits

"smaller than" and "greater than," as applied to four-, five-, or seven six-digit numbers

"expanded notation" for place-value numerals (decimal base only)

measuring lengths (metric)

measuring weight (metric)

the study of the four arithmetic operations for multi-digit numbers

the solution of "missing addend" or "missing minuend" problems such as $18 + x = 60$, or $x - 16 = 64$, etc.,[6]

experience with expressions such as $(932 - 256) \div (68 - 17)$

ability to solve word problems such as:

"1 ton of potatoes was taken from three sections: 450 kilograms from the first, and 3 times less from the second. How many kilograms of potatoes were taken from the third section?"

the order of operations, and the use of parentheses (e.g., finding the value of $72 \div 2 - 40 \div 2$)

---

[6]Here, once again, we are unexpectedly confronted by rote teaching and learning, in a highly verbal form—after some initial experience, children are to memorize the appropriate rules: "To find the unknown addend, one must subtract the known addend from the sum," and so on.

For further details on the Soviet curriculum in mathematics, please see Chapter IV of this report.

13. One of the most striking ways in which the Soviet mathematics curriculum—and Soviet teaching practice—differ from U.S. practice is this: *for the entire U.S.S.R. there is a single mathematics curriculum, specified in extremely minute detail, that is to be implemented in every classroom across the nation.* The degree of specificity will amaze U.S. observers!

So great is the uniformity that Pchelko (1971, pp. 109, 118-120), speaking to every third-grade teacher across the vast Soviet nation, can write:

> "We shall now turn to specific subjects within the curriculum and [within] the third grade textbook..."
>
> "In the second lesson, the students are given the rule for solving examples in which various operations are involved, for instance:
>
> $6 \cdot 5 + 8 \cdot 7$ $\quad$ $60 - 32 \div 2$
>
> $72 \div 2 - 40 \div 2$ $\quad$ $52 + 10 \cdot 4$"
>
> "In...the new mathematics textbooks, much attention is given to the study of the interrelationship of quantities that are in proportion to one another, [either] direct...[or] inverse... Such quantities as price, number, cost, speed, time, and distance are distinguished in the textbook, and the interrelationships between them are traced with tables (see tables on pp. 93, 101 and 104 [these references are to the *textbook* used by all third-grade teachers, and are NOT references to Pchelko's discussion of the third-grade curriculum]).
>
> [cf.] The assignment:
>
> Make up and solve problems on the basis of the following data:
>
> | Speed | Time | Distance |
> |---|---|---|
> | 12 km. per hr. | 4 hr. | x km. |
> | x km. per hr. | 4 hr.. | 48 km. |
> | 12 km. per hr. | x hr. | 40 km. |
>
> "...After considering this table, it is useful to suggest that the students solve Problem No. 787 following it with these same quantities and write it in short form in the table. ..."
>
> "[A similar] table is also given in the text [dealing] with the quantities price, number, and cost (Problem No. 896)."

*This level of specificity, in addressing all third-grade teachers in the U.S.S.R., referring to textbook problems by number, and describing exactly how every student should write his or her response, is almost unthinkable to the U.S. observer.* It implies a degree of uniformity nation-wide that the U.S. does not seek on even a city-wide basis. It implies leaving to teacher judgment no role at all in shaping the curriculum, or even individual assignments. And it implies no adjustments for individual differences between different students.

14. In the present section we have attempted to distinguish three levels of Soviet education:

a) "kindergarten" and early childhood education, for children younger than seven years of age;

b) the basic "elementary and secondary" education, provided typically in an "eight-year school" (or in the first eight years of a "ten-year school"), for children from age 7 to (about) age 17;

c) the late secondary, trade school, vocational school, junior college, or community college level (to seek U.S. parallels), which may be provided either separately, or else as the final few years in a "ten-year school" (or some roughly similar arrangement), for students (roughly) 15 years old to 19 years old.

We have attempted to give some degree of specific definition to the curriculum in the eight- or ten-year school by looking briefly at the mathematics content of the third grade. We conclude this section with some general remarks about the mathematics curriculum in the ten-year school, and some consideration of external factors which relate to basic school operation, such as teacher education, cultural imperatives, and needs of the Soviet economy.

*The Mathematics Curriculum in the Ten-year School*

Our information about the present-day mathematics curriculum in the ten-year school is, of course, sketchy. However, it seems almost certain that for younger children the program, as part of Soviet "new mathematics," became cognitively more sophisticated, somewhat resembling some U.S. programs (cf., e.g., Davydov, 1975). A ninth-grade Soviet textbook uses mathematical induction for some rather complicated proofs, suggesting at least occasionally high levels of expectation and of abstraction or formality.

From this and other observations, we have come to suspect that there may be an extreme formalization of many subjects—particularly mathematics and the sciences—in Soviet schools. In fact, Soviet educators have discussed with us the extreme difficulty of the mathematics curriculum that seems to have resulted from Soviet-style "new

mathematics." There are many reports of teachers having difficulty teaching the new curriculum. In the U.S.S.R., as in the U.S., it was realized that teachers would need special training to prepare them to teach the "new mathematics" curricula. In both countries, it seems, insufficient teacher training was actually provided. In the U.S.S.R. there are reports of large numbers of students failing (and hating) mathematics, apparently to a greater extent than formerly, although there are also reports that teachers have been working together, helping one another (and learning which sections in the books to leave out or gloss over), and that the situation is improving.

The following excerpt from an observer report is typical:

> ... in one school we visited, I saw a teacher who frankly did not understand what she was teaching. The lesson was to prove some elementary propositions in transformational geometry with respect to parallels and perpendiculars. She described to me before class what she was attempting to do. She had difficulty getting students to respond during lecture-recitation; in fact, her frustration level was high. She got only about a third of the way through the lesson and then in frustration gave the students the rest of the assignment as additional homework. It reminded me of classes I have seen in the United States. I might add this was an experienced teacher, with a very good group of students. (Romberg, 1979, pp. 9-10)

*Teacher Education in the U.S.S.R.*

Recall that the ten-year school is not necessarily divided into "elementary" and "secondary." Recall also that *all* mathematics, after grade 3, is taught by a mathematics specialist *who teaches nothing but mathematics!*

In general, teachers in the ten-year school have themselves finished the ten-year school, at about age 17. They have then received three years of teacher education at pedagogical institutes. They have almost certainly NOT attended a university. As a result, at what would correspond to U.S. elementary school levels the Soviet child is studying with a teacher who knows substantially more mathematics than his or her U.S. counterpart does, but at the higher grades this may not be true.

The situation is quite different in the various "special" schools, which we shall consider in a subsequent section of this report. Vogeli (1968, pp. 39-40) reports:

> Like the school curriculum it is designed to serve, the program for training teachers of [high school] computer programming is both intensive and of high mathematical quality. Its duration is five years, in contrast to four years for programs without dual specialization ["dual specialization" refers to high school programs that provide simultaneous specialization in

mathematics and in computer science]. Teachers for secondary schools with specialization in computer programming [and also mathematics] receive a total 4,388 hours of classroom and laboratory instruction. In contrast, graduates of four-year American colleges or universities receive about 2,000 hours of classroom instruction and laboratory work. Of the 4,388-hour Soviet total, 2,730 hours [or 62 percent of the total college program] are in mathematics, and 450 hours are in physics and electronics [so that physics, mathematics, and electronics together represent 72 percent of their entire "college" education! This is 5 times as much instruction in mathematics and physics as comparable U.S. teachers receive.].

For one of us [Robert Davis], meeting with Soviet mathematics teachers and mathematics educators had very much the feeling of the work done by PSSC, ESI, and other curriculum development groups in the late 1950's in the U.S. These were people who were very serious about the study of mathematics.

*Soviet Mathematics*

The average American may know that Soviet music achieves the highest standards of excellence (as in the work of Prokofieff and Shostakovitch), that Soviet ballet is unsurpassed in quality; that Soviet athletes are very good indeed, that Soviet chess players are not at all easy to beat. But, since mathematics looms so small on the popular culture scene, the typical American may not know the quality of advanced mathematical research in the Soviet Union. It, too, is unsurpassed. During World War II, to give one example, working independently and under conditions of strict military security, Norbert Wiener of the U.S. and Kolmogoroff of the U.S.S.R. each developed information theory, one of the major mathematical contributions to the modern world.

Soviet excellence in advanced mathematical research is so pronounced that U.S. mathematics students are regularly advised to study Russian, in order to read Russian scientific papers. In the important mathematical field of *functional analysis*, the Soviet mathematician Gelfand has been one of the world leaders. In mathematical logic, P. S. Novikov has contributed to the proof of the unsolvability of the so-called "word problem" for groups (a highly technical, very important, and quite profound piece of research in advanced mathematics). In model theory, and the relations between logic and algebra, A. Malcev (who recently died) was world famous. Within the present decade, the Soviet mathematician Y. Matiasevitch solved Hilbert's problem on the undecidability of the existence of solutions to Diophantine equations.

In short, at the level of the frontiers of advanced mathematical research, the work of Soviet mathematicians is of the very highest caliber, and of the very greatest importance.

*Soviet Teaching Style*

Given what we knew of the excellence of Soviet mathematical research, of the difficult Soviet competitive exams and their emphasis on non-routine (indeed very creative) problem solving, and on such Soviet literature as we had read, we were entirely unprepared for the teaching that we observed in Soviet classrooms. It consisted entirely, as nearly as we could judge, of rote instruction! Students are *told* that this is the hyperbolic sine (sinh x), they are *told* that its graph looks like this, and so on.

Finding this hard to believe, we inquired about it; Soviet teachers were surprised at our inquiries. Their attitude was: isn't this the way that everybody teaches mathematics? Isn't it the ONLY way to teach mathematics? Mathematical facts are facts, and if you want someone to know a fact, you *tell* him!

It appears, however, that the common practice of Soviet school teachers (and teacher belief systems in the U.S.S.R.) may be at variance with the conceptions of Soviet psychologists and educational researchers. [This may not be surprising; a similar gap exists in the U.S.] Consider, for example, the 1962 report in *Doklady APN RSFSR*, by Julius G. Goldberg, that lists 14 recommendations for effective teaching, including:

1. Teaching mathematics by demonstrating the problem solutions does not appear to be sufficiently effective. Asking students to copy ready-made solutions should not be the main method used for teaching problem solving.

2. Higher standards for mathematics teaching can be attained if the instructor discards the traditional routine, i.e., the teacher writing the solution on the board, with the students copying it down. In the teaching process, one should systematically increase the element of independence on the part of the students in accordance with the difficulty of the material and its novelty.

3. When the class is exposed to a completely new problem, the teacher should abstain from solving it himself or having one of the better students solve it at the board, so that the remainder of the students can copy the solution. It is more desirable to use a collective method whereby each student can make his contribution to the solution, and to eliminate student passivity. Students take their turns in suggesting steps for the solution; this is done either by answering the instructor's leading questions, or, without such questions, offering their own suggestions. Each suggestion is discussed by the class, the best being chosen from among the alternatives. The problem is, therefore, being solved by the entire class, and questions are put to all the students. Each subsequent step of the solution is written out on the board only after it has been elucidated to the entire class. We shall call this method frontal.

4. Frontal work should be accompanied by board-work only in the case where the problem is very difficult, or if the students are not yet familiar with the form of its written presentation. If a problem is difficult, even after a given step has been discussed and clarified, the teacher cannot be sure that the students will carry out the solution correctly. At this point, in order to verify the correctness of separate steps in the solution, one may resort to writing the material on the board. Care should be taken as to the proper timing in this sequence. The presentation on the board must appear at a point in time somewhat later than when the material is written by the students, themselves, in their notebooks. As the students master the solutions to given types of problems, we can dispense with the board presentation of separate, easier steps, gradually shifting to frontal work without board presentations. At the teacher's request, the students put forward their suggestions: these are discussed and perfected; and finally, the students write them down.

....

6. The teacher's ultimate goal is to teach his students to be able to solve problems on their own. Therefore, all types of frontal and semifrontal work should alternate with partially and totally unassisted work. The usual order, while introducing a new problem type, would normally be: begin with frontal work, pass on to semifrontal, then through partially unassisted to completely unassisted.

....

....

9. When a new chapter is started, students should be given a set of exercises apart from the daily homework, covering all the material from the forthcoming chapter. The students do these problems as they progress through the chapter and turn them in at the end of the time allotted for the chapter. As a rule, this set should contain more challenging problems, and the students should have more time at their disposal to fully think over the problem. They are often deprived of such an opportunity when doing daily assignments. Moreover, daily work is normally assigned from the text, where the title of the section prompts the way to the solution. But, as soon as there is a way to solve the problem, there is no longer a problem. Review exercises may contain textbook problems, but without reference to their section. Likewise, when solving problems in class, the number of the problem and number of the section can be mentioned only after the problem has been solved. The students are well aware of the difficulties that this practice entails, but they are also aware of the benefits. In the case of teaching calculus, for instance, it is especially true of problems involving techniques of integration, convergence and divergence

of infinite series, multiple integrals, differential equations as well as applied problems.

...

11. As far as daily homework assignments are concerned, it is useful to discuss the more difficult problems in advance. This should be done when the teacher realizes that the students are unable to cope with the problem on their own during the limited period of time before the next class. Needless to say, one is not compelled to assign such problems at all. However, it is beneficial to expose students to more challenging problems gradually, discussing certain points with them, and at the same time giving them an opportunity to try and carry out the solution on their own. Special emphasis should be placed on the discussion of those problems which develop an understanding of the material in general. A preliminary discussion is indispensible for problems and questions which give the fundamental concepts of the course, for example, in calculus: function, limit, derivative, differential, integral, continuity, convergence, the mean-value theorems, basic facts about vector space and the real-number system. [Goldberg, 1962; 1978, pp. 385-388]

*Educational Television in the U.S.S.R.*

American visitors in their hotel rooms, seeking entertainment, may find that the television set will bring them mainly the same rote lecture on the hyperbolic sine that they could observe in living color in various school classrooms. We do not have statistics on how much Soviet TV broadcasting is devoted to such lectures; by our very small sample technique, we consider that it might be quite a sizeable proportion.

*Student Response*

Given the warm and caring school, the formal, academic curriculum, the rote instruction, the cooperation of the collective, and the pressure to take one's work seriously, how do Soviet youngsters respond? One observer reports:

In spite of many reservations I have about the lack of variability of instruction and the lack of attention to individual differences, it was apparent that the students were engaged during instruction. I did a random time sampling in one 50-minute mathematics class. I estimated that for 37 of the 50 minutes of instruction, students were actively engaged in learning. This is considerably higher than any math classes I have observed in [the U.S.]...In particular, there was much less wait-time and transition-time than I have found in Ameri-

can classes. This is partly due to the pedagogical structure; i.e., [mainly] lecture-recitation...Also, there is very little seat work or work with manipulatives. (Romberg, 1979, p. 8)

*Equipment in Soviet Schools*

There are essentially no computers in Soviet schools, nor hand-held calculators. But the schools we visited were remarkably well equipped with other materials. In a social studies class one would find videotape recorders being used to present material via TV monitors. A special 16 mm projector, *designed by educators*, is said to be in every school in the U.S.S.R. We did NOT, however, observe any Soviet equivalents for Cuisenaire rods, geoboards, or Dienes MAB blocks, which play an important role in many U.S. schools.

*Vospitaniyeh* [ВОСПИТАНИЕ]

There seems to be no English equivalent of this word. It means "moral education," "the inculcation of good work habits," "teaching good citizenship," "the development of a sense of responsibility." This is VERY prominent in educational thought in the U.S.S.R. U.S. public schools have, today, no equivalent, although in the McGuffy readers they perhaps once did have. Various U.S. private and religious schools do provide equivalents, however.

*"How To Be A Parent."*

Soviet educational philosophy advocates leaving NOTHING to chance. This includes the question of how parents should behave. There are many books on "how to be a parent," and these are widely read and much discussed. There is a single official Soviet theory on bringing up children—very, very different from the myriad competing theories in the U.S. In the U.S.S.R., deviant theories are denounced, forcefully.

*Adult Attention to Children*

Soviet and U.S. practice diverge dramatically on the matter of supervision of children. By building large schools, staffed by few adults, the U.S. in effect creates teenage ghettoes. The abundance of rock music and other aspects of teenage culture has created a teenage world where adults are uncomprehending strangers. (Ask a teenager to explain the background music in the movie *Coming Home*, and compare what they say with your own spontaneous perceptions!)

The Russians think children should be brought up by adults, supervised by adults, watched over carefully AND CLOSELY by adults. U.S. families in Moscow are always finding themselves accused of neglecting their children. As our example, Soviet grandmothers wait outside schools to walk home with their grandchildren. The U.S. system of having a group

of children walk together under a peer leader seems to the Soviets to be a shameful neglect of adult supervisory responsibilities. And letting children walk home from school by themselves (!)—as is done in Urbana, Illinois, in what is probably the typical U.S. pattern—would be entirely unthinkable. Scandalous child-neglect!

*Caste Systems and Access to Schools*

Hedrick Smith reports that the separation in the U.S.S.R. between intellectuals and blue-collar workers is absolute and is hereditary. Children of blue-collar workers do NOT play with the children of white-collar workers. And they know, absolutely, what they'll be when they grow up.

From an observer report is this comment:

> In spite of the rhetoric about a classless society, there is very strong evidence of cultural and social class differences. The Estonians made a point of describing the unique Estonian aspects of their program. Their educational system is not identical to that in Moscow. The Soviets also admitted there were problems in rural areas in contrast to city education. And finally, in the English-speaking school we visited in Leningrad, we were told that children of all segments of society had the opportunity to attend such a school...However, we met and talked only with children whose parents were in professional positions. (Romberg, 1979, p. 10)

*Control of the Culture*

Soviet theory and practice value, and aim for, a high degree of control of the culture. This is generally well-known in relation to plays, movies, operas, etc., which present approved views of approved topics handled in an approved way. Similar is the case of poetry (which is very important in Russia). Painting is also similar. The Soviets heavily subsidize Prokofieff, Mozart, and Beethoven, as well as Russian folk music, but rock is anathema. So are some western dress styles. Television presents what is good for Soviet culture and Soviet society, and NOTHING else.

The extent of this was brought home to us when we met with members of a committee that approves toys. When a toy is proposed, this committee studies it carefully. Until and unless the committee concludes that the toy will teach children desirable behaviors or values or expectations, and will NOT teach undesirable ones, it will not approve the toy. *And without the approval of this committee, that toy cannot be manufactured.*

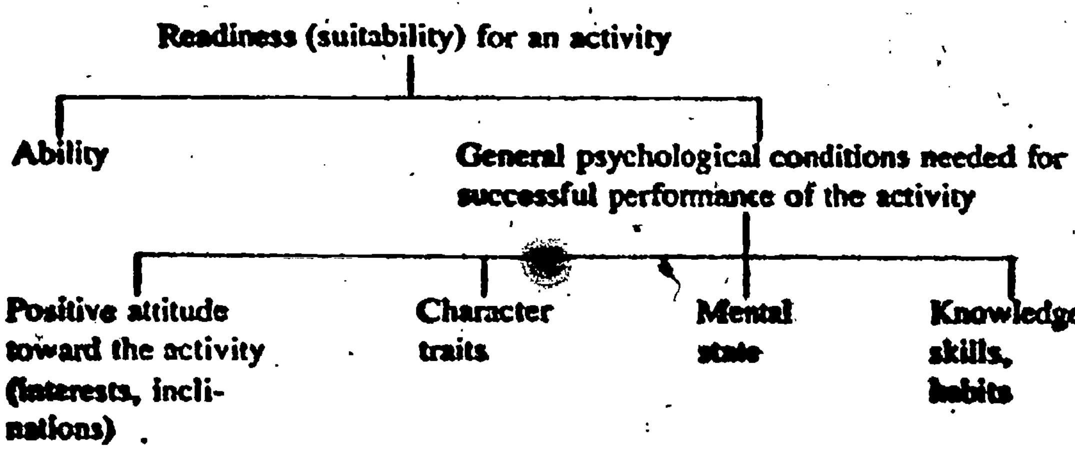
*Is Everyone the Same?*

A great deal of Soviet thought is based on the premise that everyone is really the same. Yet, to this thesis there is an antithesis:

Soviet work on defectology, for example, is regarded as excellent—this corresponds roughly to "special education" in the U.S. In the following section of this report we shall consider special Soviet schools for the mathematically gifted, with their highly selective entrance examinations. In the Soviet Union we had the opportunity to meet with psychologists whose task it is to find which people should become truck drivers, which people should become teachers, which people should become astronauts, and so on. These psychologists were clearly humane people who thought that proper placement could greatly add to the sum total of human happiness.

Soviet thought tries to separate "ability" to do mathematics from the "inclination" to do mathematics, as this excerpt from Krutetskii (1976, p. 74) indicates:

> The entire "ensemble," a synthesis of personality traits, since it is a considerably broader concept than ability, we prefer to call *suitably* or *readiness for an activity*. Anan'ev, for instance, uses the idea of "readiness for highly productive activity in a definite province of work, of societal life" (20, pp. 16-17; 21, p. 15). Rubinstein uses the idea of "suitability" (354, p. 533; 355, p. 126), although he connects it with the concept of ability proper, rather than with the concept of the "ensemble." A. V. Yarmolenko (474, p. 78) cites an interesting instance of the divergence of interests (inclinations) and abilities. Student M, in the opinion of all of his teachers, possessed very strong abilities in mathematics but openly hated it. Since Student M did not combine abilities, properly speaking, with inclination, Yarmolenko concluded that Student M had "no real ability, but only an external image of it, which led to a false diagnosis." We would have put it differently: Student M had ability, but there was really no readiness or suitability. It was not a case of "false diagnosis." It would have been, however, if one had said that Student M was suited for mathematical activity. Our diagnosis is that he was capable but not suited, due to the total absence of inclination.
>
> The above interpretation of the structure of suitability (readiness) for activity is represented schematically below:



Thus, suitability for an activity (the ability to perform it successfully, or ability in the broad sense) cannot be influenced only by the presence of ability in the proper sense. It can be influenced only by a whole complex of essential personality traits, involving the intellectual, the emotional, and the volitional.

Thus we can define the concept that is fundamental to our study. By ability to learn mathematics we mean *individual psychological characteristics (primarily characteristics of mental activity) that answer the requirements of school mathematical activity and that influence, all other conditions being equal, success in the creative mastery of mathematics as a school subject—in particular, a relatively rapid, easy, and thorough mastery of knowledge, skills, and habits in mathematics.* The specific content of the concept of mathematical ability will be disclosed later, when we analyze its structure.

D. N. Bogoyavlenskii and N. A. Menchinskaya, in discussing individual differences in children's educability, introduce the idea of psychological properties that determine success in studies, all other conditions being equal (44, p. 182). They do not use the term "ability," but their idea is essentially close to the definition given above. And if one speaks of educability, then ability to learn mathematics is, in our interpretation, apparently none other than the property of creative educability in mathematics.

Krutetskii has much more to report on individual differences in abilities, skills, and inclination, but that leads us into Soviet theoretical thought, rather than classroom practice. For the present we merely note that the tension between an axiomatic equality of individuals and a pragmatic selectivity shape actual educational practice at every level, and make for special complications in the rhetoric that is employed.

### *Needs of the Soviet Economy*

Analyses seem to indicate that the Soviet economy suffers from a shortage of skilled workers at nearly every level. European firms, such as Fiat, have difficulty finding workers with adequate measurement skills and skill in shop mathematics. We have heard laments over the Soviet shortage of computer experts. This seems to represent a marked difference from the situation in the United States—and it is surprising, given the goals of Soviet education and the emphasis which seems to be placed on computer science.

## C. *Special Schools*

In discussing the basic "common" school, roughly akin to U.S. elementary and secondary public schools (the Russian "eight-year school" or "ten-year school"), the Soviet egalitarian theme comes through very forcefully, at least working from the sources presently available to us.

But, in any nation with the ballet, music, athletic, chess, scientific, and mathematical performance of the U.S.S.R., there MUST be an opposing theme, a "quality" or "excellence" theme—and, indeed, there is. In this section, building primarily on the experience and studies of Bruce Vogeli, we report on the "special" schools for mathematically talented students, and the special high schools that educate future computer scientists. In looking at these schools, we see a concern for quality or excellence that, in its direction, goes as far beyond typical U.S. practice as the Soviet egalitarian approach does in the opposite direction. We are dealing here, unmistakably, with a talented elite, who are to be recognized as such, treated as such, and educated as such—at public expense. Nearly all U.S. parallels (of which there are, in any case, very few) are private, and stand outside of our publicly-supported systems.

Imagine identifying students gifted in a specific discipline, selecting them out, and having them leave home, to attend a special *residential* high school for students who share this *specific* gift! That is what we find when we look at the "special" Soviet schools.

The preceding Section B might be described as a long-distance macroscopic view of the Soviet school, through a wide-angle lens. We see a single type of school, perhaps not even divided into our familiar sub-categories of "elementary" and "secondary."

When one looks in more detail, to be sure, one sees some very important variants—variants more extreme than one finds among public schools in the U.S. There are, for example, *special residential schools for mathematically gifted students*. This is a more extreme "special treatment" than any in the U.S. There are, however, four such schools in all of the Soviet Union [or were at the time of the Vogeli study (1968)].

Our big, broad picture cannot see such small details. (We do not mean to minimize the importance of these schools, nor of other Soviet variations. What we mean to emphasize is an *almost* total homogenity, even identity, of Soviet schools, with no variation—except for a very small number of special cases.)

Setting aside the infant schools and the Pioneer Palaces, what kinds of Soviet schools are of importance for mathematics education? The answer seems to be as follows:

a) The "ten-year school," representing the schooling for almost every Soviet child, attended from age 7 through age 16,

perhaps with no division into "elementary" and "secondary," which we considered in Section B.

b) Boarding schools offering a specialization in mathematics. There are four of these in the Soviet Union; each is sponsored either by a university or by the Academy of Science. Their affiliation is, in U.S. terms, more with the College of Arts and Sciences than with a College of Education. A. N. Kolmogoroff (one of the greatest mathematicians of the twentieth century) teaches at one of them three days a week (or did, as reported to the Moscow Mathematical Society in March, 1964; cf. Vogeli, 1968).

The seriousness of intent involved in operating boarding schools is notable, a typical instance of Soviet determination. Kolmogoroff's Moscow school includes grades 9, 10, and 11 in the Soviet system (ages 15 through 18.) Instruction in mathematics and physics occupies on the average 17 hours per week, or 51 percent of the total number of 33 hours per week. The overall curriculum, in terms of time allocation, looks like this:

| | |
|---|---|
| Russian Language and Literature | 9% |
| History and Social Studies | 10% |
| Geography | 4% |
| Biology | 2% |
| Chemistry | 6% |
| Foreign Language | 9% |
| Physical Education | 9% |
| Mathematics and Physics | 51% |

The curriculum in mathematics and physics is divided up as follows:

| | |
|---|---|
| Algebra and Analysis | 23% |
| Geometry | 17% |
| Linear Algebra | 5% |
| Discrete Mathematics | 7% |
| Probability Theory | 3% |
| Problem Seminar | 9% |
| Physics | 37% |

Vogeli (1968, p. 51) reported:

> Because the Moscow school is a boarding school, Kolmogorov is able to stimulate individual pupil research and evening discussions relating to mathematics to a far greater extent [illegible] is possible in day schools.
>
> ...Boarding-school pupils study functions of a complex variable in analysis and the elements of a group theory in algebra in some detail...

c) Secondary day schools with specialization in computer programming. As of 1968, Vogeli reported that there were about 100 special secondary day schools in the Soviet Union that offered a specialization in computer programming.

d) English-speaking schools. Another special kind of school, of interest to Americans, is the English-speaking school. We do not know how many of these there are, but it is clear (because we have visited some of them personally) that they do succeed in producing students with a good command of English. Beginning in the first grade (chronological age 7 years) the students study English, and by age 15 or 16 the students are *taught in English* in most of their courses.

e) Part-time schools for mathematically-talented students. These schools meet *outside of regular* school hours, in some cases meeting evenings. Admission is less restricted than admission to the boarding schools. In one such school in Ivanovo, classes meet twice a week, for two- or three-hour sessions (Vogeli, 1968, p. 55).

f) The mathematics correspondence school. Established in 1964 at Moscow State University, the "Republic Mathematics Correspondence School" was intended to serve mathematically talented youth throughout the entire Russian Republic (*not* the entire Soviet Union!). The intent was to reach students who could not be accommodated in day, boarding, or part-time schools. Vogeli (1968, p. 63) reported:

> Correspondence pupils from the same school or community are permitted to complete lessons collectively. In the opinion of school officials, "nothing but good can come from collective work—in the first place, ...groups [of pupils who work collectively] can be formed by degrees into a first-class school circle [a math club, especially one that specializes in coaching for the highly demanding Soviet "mathematics Olympiads"]; and, in the second place, at the present time in science basic strength rests, not with the individual, but with the group."

Vogeli also reproduced the entrance examination for the Correspondence School for 1964. Because it gives an indication of the flavor of Soviet work, we reproduce it here:

1. Two people play the following game: the first names a one-digit number (that is, a number from 1 through 9); the second adds to it another one-digit number and states the sum. To this sum the first player adds still another one-digit number and states the sum, and so on. The winner is the one to name 66 first. How should one play this game in order to win? Who is the winner in a fair game of this sort, the beginning player or his opponent?

2. Factor:

a) $x^8 + x^4 + 1$ (into 3 factors)

b) $x^5 + x + 1$ (into 2 factors)

3. From the vertex B of the triangle $ABC$ draw the median and the altitude. Assume that they divide $\angle ABC$ into three equal parts. Determine the measures of the angles of $\Delta ABC$.

4. Four children—A, B, C, and D—participate in a race. After the race, each asks the other in what position he finished. A answers, "I wasn't first and I wasn't last." B answers, "I wasn't last." C answers, "I was first." D answers, "I was last." Three of the children have answered honestly, but one has not. Which one has answered incorrectly? Who was first?

5. How many six-digit numbers are there, all digits of which are odd?

6. Prove that in an arbitrary triangle

a) the sum of the lengths of the medians is less than the perimeter, and

b) the sum of the lengths of the medians is more than 3/4 of the perimeter.

7. On a table lie some books that must be wrapped. If they are wrapped four, five, or six to a package, then one extra book remains; but if they are wrapped seven to a package, then there are no extras. How many books could there be on the table?

8. Construct a triangle given two of its sides, $a$ and $b$, if it is known that the angle opposite one of them is three times greater than the angle opposite the other.

9. a) Find all numbers satisfying the equation

$$x + y = xy.$$

b) What kind of positive whole numbers could satisfy the equation

$$x + y + z = xyz?$$

10. A four-digit number is multiplied by the four-digit number obtained by writing its digits in opposite order. The eight-digit number obtained has zeros as its last three digits. Find all such four-digit numbers.

11. a) Construct the circle tangent to both a given circle at a given point and a given line.

    b) Construct the circle tangent to both a given circle and a given line at a given point on the line.

12. a) How many roots does the following equation have?

    $$x^2 - 3x + 1 = 0$$

    b) Sketch the graph of $y = x^2 - 3x + 1$.

Many observers have commented on the contrast between Soviet and U.S. demographics, in music, athletics, science, mathematics, and other fields. At the highest levels of excellence, on a world-wide basis of comparison, one finds Sergei Prokofieff one of the very greatest composers of the twentieth century (and probably of all time); David Oistrakh, one of the greatest violinists of all time; Rostropovitch, one of the greatest cellists of all time; and so on. The Moscow Symphony is one of the world's greatest. Soviet ballet is unexcelled, possibly unequaled. It is the same in many fields of endeavor, including (emphatically!) mathematics.

But one is talking about a small quality elite. The U.S. has not merely the N.Y. Philharmonic, but also the Boston Symphony, the Philadelphia Symphony, the Pittsburgh Symphony, the Chicago Symphony, the Cleveland Symphony, the Los Angeles Philharmonic, the St. Louis Symphony, and so on, for a VERY long list.

In athletic terms, the Soviet first team is fully a match for anyone, but they tend to lack depth on the bench and in the bull-pen.

There is little to be gained, and much to be lost, by comparing national systems in an "our-system-can-beat-your-system" frame of mind. The important gains are to be had only if each nation tries to *learn* from the other. This is not unreasonable—no country has an educational system that satisfies all the national needs, and all the personal needs of individual citizens. In that sense, every nation should be seeking improvement. One road toward that improvement may be the careful study of what other nations do, in the hope of learning from them.

In that spirit, the U.S. can learn some valuable lessons from studying the "special" schools in the Soviet Union.

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# Chapter III

## Difficulties in Language

Robert B. Davis

Precise communication between Soviet and U.S. researchers is severely impeded by differences in language that reflect deep underlying differences in concepts, assumptions, and expectations.

Every Soviet school has its own full-time physician. But is this "physician" really similar to a "physician" in the U.S. sense of the word, or more nearly a "nurse practitioner," or is she (most are female) neither? How could we describe her in U.S. terms?

We have already considered the Russian word *vospitaniyeh*. It has no English equivalent as a word, because it has no U.S. equivalent as a *practice* or as an *idea*. *Within* some special religious groups it may have a parallel. It combines moral education, the inculcation of good work habits, training to live within the collective, developing a sense of responsibility, shaping good citizenship, forming values, aspirations, and self-concepts, and holding up desirable examples for emulation. It includes parent education, and is a major component in the professional training of early childhood specialists ("upbringers").

We had a demonstration of translation complexities when we met two psychologists who, according to the translator, were specialists in "genetic psychology." What did *that* mean? Piaget has called his work "genetic epistemology"—was *that* what was meant? But in recent years the Soviets have been generally negative toward Piaget.[1] Could they possibly mean Piagetian work? Only by very great insistence—almost to the point of being obstructionistic—was it possible to discover what they meant by "genetic psychology": they studied the differences in abilities and inclinations of identical twins!

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[1]The Soviet attitude toward Piaget is somewhat similar to their attitude toward Tolstoi, which Menchinskaya (1969) expresses as follows:

It was once contended that the teacher should not interfere with the child's process of conceptualization. L. N. Tolstoi expressed this very clearly. He interpreted the formation of new concepts as a "complicated, mysterious and delicate process of the spirit," with respect to which "any interference is a rude and awkward force delaying the developmental process"...On the question of concept development, Tolstoi essentially held to the false theory of free upbringing. He insisted that "the school provides no new concepts" and that "only first-hand, vital relations with objects can teach and provide new concepts..." He also stated that, "Freedom is the most advantageous condition for acquiring the greatest number of concepts..." In Tolstoi's view, the school can

We have also considered the case of the "collective," a dominant influence in the life of every Soviet citizen, for which there is no equivalent in the experience of most Americans.

Some idea of the complexity of the translation problem can be seen in this example from Krutetskii (1976, pp. 76-77):

> Of all the most widely used foreign scientific terms corresponding to the Russian *sposobnost'* (ability), the German *Fähigkeit* comes closest to our interpretation. Unfortunately, we can accept none of the three English terms that denote the idea of *sposobnost'* as adequate to our concept of ability without essential reservations. In English and American psychological and educational literature, the terms *ability*, *capacity*, and *aptitude* are used, each of which has its nuances. *Ability* is something midway between our concepts of *sposobnost'* and *umenie* (skill)(it means skill in performing an operation, including the solution of mental problems), although there are special terms—*attainment*, *acquirements*, *knowledge*—to designate analogues of the Russian terms *priobretenie*, *znanie*, *umenie*.* Teplov was even inclined to believe that *ability* does not mean ability (*sposobnost'*) at all but designates an aggregate of habits and skills (408, p. 11). The aspect of the organism's innate possibilities is too heavily stressed in the term *capacity*. The third term, *aptitude*, is less widely used and seems to convey the meaning of *ability*, but it connotes inclination for an activity. *Sposobnost'* is usually translated into English as *ability*.
>
> Mathematical ability (as, in general, all abilities for complex types of activity) is a mental formation that is complex in structure. It is a unique synthesis of properties, an integral quality of the mind, including diverse mental aspects and developed during the process of mathematical activity. This aggregate is a unique, distinct whole: only for purposes of analysis do we single out individual components, by no means

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only systematize concepts that are unconsciously acquired in the process of living. It is perfectly clear that such a view fundamentally contradicts the basis of Soviet schools. In our schools the children consciously master the "foundations of the sciences," and the teacher directs this process.

It was not arbitrarily, then, that Soviet psychologists substituted the term *mastering* for the terms *conceptualization* and *development* as they applied to teaching children scientific concepts. A. A. Smirnov uses the term *mastering* in his article, "Questions on the Psychology of Concept Mastery in Schoolchildren..." That article is, in fact, the first psychological work to characterize the process of concept acquisition in schoolchildren and the changes which the children undergo as the result of instruction. Such a formulation of the question raises a whole series of research problems. (pp. 76-77)

regarding them as isolated properties. These components are closely connected, influence each other, and form in their aggregate a single system, whose manifestation we conventionally call the *mathematical giftedness syndrome* (a number of interrelated elements that characterize the psychological phenomenon).

The term "giftedness" was current in Soviet psychology in the forties and fifties. Teplov (409, 411, 412, 413) convincingly showed that to implement any more-or-less complex activity one needs not one ability but a whole series of them. The unique combination of abilities on which the possibility of performing an activity successfully depends Teplov called giftedness in the activity. As he himself indicated, the only reason for designating an aggregate of inclinations by the term "giftedness" is the literal meaning of the Russian word "giftedness" (*odarennost'—darovitost'* [giftedness]—*darovanie* [gift, talent]—*dar* [gift]—that is, what is given from birth). After Teplov, other authors of textbooks and manuals adopted a new terminology. P. A. Rudik interpreted "giftedness" as "innate characteristics which are preconditions for the development of abilities" (359, p. 397); P. I. Ivanov used "giftedness" to mean "inborn traits which, as they develop, manifest themselves in abilities" (139, p. 423); N. D. Levitov defined it as "an innate fund of abilities that are anatomical or psysiological inclinations" (249, p. 82).

We feel that at the contemporary level of development of psychology, this last definition of "giftedness" tends to be meaningless. We still do not know which specific anatomical or physiological features are inclinations for mathematical ability, and the term "giftedness" (in the sense of an aggregate of inclinations) remains empty. Perhaps this is why some leading Leningrad psychologists react negatively to the concept of an inclination in its anatomical or physiological sense, asserting that it is "only a logical conjecture . . . a verbal screen for unknown causes" (Kovalev and Myasishchev, 174, p. 63). Therefore we have found it sensible to return to the former meaning of the term "giftedness." Mathematical giftedness is the name we shall give to a unique aggregate of mathematical abilities that opens up the possibility of successful performance in mathematical activity (or, with schoolchildren in mind, the possibility of a creative mastery of the subject).

As a further example, we have mentioned our surprise at the rote presentation style in Soviet mathematics classes which we observed. An American can easily form wrong images when hearing the word "teaching" as applied to mathematics classes in the U.S.S.R. Presumably Russians have a similar problem when *we* speak of "teaching mathematics." They might be quite surprised to see what we actually do.

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*Krutetskii gives his translations into English of the Russian words here. A closer approximation of the usual rendering into English would be *acquirement, knowledge, skill* or *acquirement, achievement, skill* for, respectively, the Russian *priobretenie, znanie, umenie*.—ED.

The problem of confusion of language has long plagued education, in a most extreme fashion. Soviet-U.S. differences merely exacerbate a fundamental difficulty. Looking only *within* the U.S., one finds nearly incredible levels of confusion. "Discovery" teaching has meant so many entirely different things as to be virtually meaningless, or worse (cf. Davis, 1967).

The term "new math" has sometimes referred to abstract mathematics; at other times it referred to highly "concrete" mathematics using geo-boards, Cuisenaire rods, Dienes' MAB blocks, and other materials. It has referred to Euclidean geometry, to Cartesian analytic geometry, to "informal" geometry, to engineering drawings and descriptive geometry, to vector geometry, to transformational geometry, and even to "concrete" geometry, as in Marion Walter's work with milk cartons.

*This level of linguistic confusion is virtually an ABSOLUTE barrier to real educational progress. It is literally impossible, in most cases, to tell what someone is talking about.*

*Serious Soviet-U.S. communication MUST use very specific examples, sample test items, films or videotapes showing specific lessons, and similar specific items, in order to give adequate definition to the words that are being used. [Actually, this is equally necessary within the U.S., but that should be part of a separate report.]*

One "new math" controversy relates to the propriety, or otherwise, of "teaching sets" in the primary grades. So wide a range of activities is included by different teachers and educators when they speak of "teaching sets," that the positive or negative value of this ill-defined activity cannot possibly be determined. Moreover, the *identical* activity will be described as "teaching sets" by one observer, and as "not teaching sets" by another observer. This is the level of confusion that exists if one looks merely within the United States!

In going through existing English translations of Soviet writing, we have found the following, which (unfortunately) is entirely typical:[2]

> [Conclusion of a research report]: "The study made it possible to conclude that fairly complex forms of analysis and synthesis of the properties of perceived objects, comparison and generalization of observed phenomena, and understanding of the simplest relationships and their interaction could be formulated in children."

[2]We omit specific citations since we do not mean to be critical of any particular writer or translator. The problem, regrettably (or alarmingly), is an entirely general one.

What, exactly, does *that* mean? Or consider this example:

> [From an article advising Soviet teachers how to teach the Soviet version of "new math"]: "However, one should not reduce all work on the study of operations to nothing more than the solution of examples and problems. Skills must be based on the properties of operations only under comprehended conditions so that they are well understood and hence lasting. The successful formation of skills also assumes knowledge of the interrelationship of operations and the interrelationship of components and results of operations. There thus follows the necessity for including these matters in the work plan for the study of operations."

Here again one finds oneself asking: "What, exactly does that mean? What will the teachers assume that it means? How might one expect the teachers' behaviors to change as a result of this 'official' advice from an expert?"

In most such cases the reader can make an educated guess—but this comes closer to projective tests like the Rorschachs and the TAT than to serious scholarly discussion.

Every effort must be made to determine, as precisely as possible: EXACTLY what was done? EXACTLY what was intended? How, EXACTLY, did students perform?...and so on. This will be no small task, but without it there will be little progress and great confusion.

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# Chapter IV

## A More Detailed Look at Soviet Mathematics Curricula

Robert B. Davis

The parallels between Soviet efforts to improve and modernize school mathematics and similar U.S. efforts are so striking as to deserve a more detailed consideration. In this chapter some of the details of Soviet curricula will be considered, including changes that have been attempted from time to time.

If we begin around 1921, with the published mathematics syllabus of the so-called "Unified Labor School," we find, at the outset, a considerable surprise. The Soviet program of 1921 bore some considerable resemblance to the U.S. "new math" programs of the 1950's and 1960's! Vogeli (1971, p. 6) reported that "the study of arithmetic was compressed into four years," that "plane and solid geometry were partially integrated," that the function concept was emphasized as a unifying theme, and that the eighth- and ninth-grade program included parts of both analytic geometry and introductory calculus!

Compare this with the U.S. "new math": spending (at least somewhat) less time on arithmetic was a theme of several "new math" projects (including the Illinois Arithmetic Project of David Page and the Madison Project of Syracuse University and Webster College); combining plane and solid geometry was a recommendation of the early CEEB Commission on Mathematics reports of the 1950's, and continues to arise as an alternative treatment for geometry (but somehow this is a change that is made over and over again, rather like giving up smoking, without ever being finally accomplished; in 1979 there were still schools in the U.S. adopting this innovation); the Madison Project, among others, used the concept of *function* as a main unifying theme in grades 4-12 and introduced analytic geometry in these grades; and the excellent textbooks by Stein and Crabill (1972) introduce beginning calculus in grades 8 or 9 (at University High School in Urbana, Illinois, these texts are used to introduce calculus in grade 7, so that tools such as differentiation are available throughout the entire secondary school program). The similarities are indeed striking and not unimportant ones. They represent a major re-shaping of school mathematics, particularly with respect to the inclusion of functions, analytic geometry, and calculus *early* in the program.

The number of times this program has been (independently) designed by experts is testimony to its internal strength—internally, it can be a very effective curriculum. The number of students who have successfully been educated by this curriculum is large, and provide evidence that it *can* be a feasible program (Davis, 1976a, 1976b; Hannibal, 1976). But experience in Russia in the 1920's and in the U.S. in the 1960's

demonstrated--if there were ever any doubt--that ambitious curricula of this type require special (and extensive) teacher education, and that they are appropriate for only *some* students, almost certainly not for *all* students.

In Russia in the 1920's, Vogeli (1971) reports that "in actual practice the 1921 syllabus proved too difficult for general use, and a less demanding syllabus--known as the minimal syllabus--was employed by most schools" (p. 6). But surely the essential questions really are: "For *which* students is such a curriculum appropriate?" and "How can one provide adequate teacher education?"

Of course, in the Soviet Union the 1921 syllabus had the additional important disadvantage of operating in a society that was being transformed by a revolution and that was devastated by war. Furthermore, the nation was being remade economically, with an unprecedented emphasis on industrialization for which Russia's peasant past had not prepared it. Soviet schooling was reoriented toward practical vocational roles in industrialization. As a result there was a decline in mathematical scholarship (Vogeli, 1971, p. 6), and the school mathematics syllabus lost much of its theoretical content. The new goal was to train students in immediately useful practical skills of a low-level vocational sort.

(Vogeli brings up another matter that deserves study--the Soviets embarked on what Vogeli calls "the weakly-constructed complex-theme curriculum" and began to employ the "brigade" or "project" form of teaching. It would be interesting--if it is still possible--to find out what this actually meant, as it was implemented in the 1920's.)

In any event, by the end of the 1920's the graduates of the unified labor schools were judged to be too weak in theoretical mathematics to continue easily into university-level studies, and too weak in practical skills to be employable in factory jobs. Under the circumstances of the urgent Soviet industrialization, this situation was considered unacceptable--something had to be done!

After discussions at the 1931 and 1932 Communist Party Congresses, it was decided to rearrange the entire structure of Soviet schooling. An immediate goal was the creation of a vast army of skilled industrial workers. The "brigade" and "project" teaching methods were abandoned, and in their place a very formal academic program was reinstated, presented via highly-structured lessons (a pattern that one seems to see in Russian schools today). Old mathematics textbooks by Kiselev, which had been discarded, were now resurrected and put into use.

The 1932 program remained in use for twenty years, until in 1953 the Ministry of Education of the Russian Republic proclaimed the need for modernization. The new goal was to provide more in the way of polytechnic labor training, via what is sometimes called the "Kruschev school reform" (Shabanowitz, 1978, p. 40). The central problem, as described by Shabanowitz, was that the demographic mix of skills and

career goals that was being created by the Soviet educational system did not match the demographic mix that was needed by the Soviet economy. Russia had been moving very rapidly from a peasant society to a modern industrialized society. Where there had been one-year schools in peasant villages that attempted to teach "addition, subtraction, multiplication, and division" as effectively as they could within the program of a single year of schooling, there were now eight-year schools that roughly matched the U.S. K-12 schools. Russian youth were suddenly far more sophisticated than in former years, and very large numbers of them sought places in higher education.

A typical U.S. response would probably have been a vast effort to expand the institutions of higher education, with all that that might entail. The Soviet response was also typical: it was decided, first, that a rapid expansion of higher education would necessarily imply a lessening of quality standards; second, that the Soviet economy *did* indeed require large numbers of increasingly more sophisticated workers, but at something considerably less than the university level of sophistication. It was decided to "strengthen the ties between school and life." The program to do this was called "polytechnical labor training," and was intended to prepare young people to take up a vocation immediately after graduation from school. Students were to participate in socially useful labor at an early age so that they could plan their careers on the basis of more mature personal viewpoints, and so that they would not automatically look to higher education as the only possible next step after high school. For admission to institutions of higher education, preference was to be given to applicants who had several years of practical work experience after high school, as against applicants who sought to enter college immediately upon completing high school.

(In both the U.S. and the U.S.S.R. there is a persistent theme of the unresolved tension over more or less education, and the resultant effect on lifetime careers. You can quickly learn to be a key-punch operator, a typist, a draftsman, or a practical nurse. Considerably more education is required to become a computer scientist, a management consultant, an engineer, or a surgeon. But the lifetime rewards, at least in economic terms, will be far greater. In both the U.S. and the U.S.S.R., many parents appear to suspect that if their children are being prepared for a career immediately after high school, it will be a career with a low ceiling for long-term economic prospects--in effect, they feel, their children are being shanghaied into a lifetime of menial labor. The issue is unresolved within the U.S., and one presumes also within the U.S.S.R.)

In effect (if one seeks a very rough U.S. equivalent), by the Kruschev plan the Soviets created a program that combined the following U.S. institutions: the vocational high school, the technical institute, the community college, the Civilian Conservation Corps, the Future Farmers of America, and the draft (but for work, not military service). Discussions on the Kruschev revision took place from 1953 until 1958, by which time the reorganization of all Soviet schools was proclaimed by the Supreme Soviet of the U.S.S.R.

Perhaps unsurprisingly, action and reaction were equal and opposite, though they may have taken subtly different forms. Faced with the prospect of more low-level vocational and technical emphasis, representatives of the humanities were able to get statements reaffirming the value of the humanities, and promising that "there must be no reduction in the humanities as a result of the school reorganization" ("The Party...", 1958, p. 43).

Confronted with the prospect of mathematically gifted students running off to farms to gather crops instead of learning about conic sections, or working in a factory instead of studying the delta process, the scientists and mathematicians were presumably in an even stronger position to fight back. A group of scientists and mathematicians petitioned the Communist Party leadership and the Ministry of Education to establish courses in theoretical mathematics in special schools for mathematically gifted students. Thus were the "special schools" initially created.

Of course, in a way, studying calculus and physics and biology in high school *is*—or at least, can be—preparation for the world of work, if one is to go on and become a computer scientist, a surgeon, or an aeronautical engineer.

During the 1960's, the role of the special schools increased rapidly: by 1965 more than 100 such schools were in operation, enrolling altogether more than 25,000 gifted or talented students. For the nation as a whole, however, this was still a matter that involved relatively few students.

That, however, was to change. The more sophisticated mathematical curriculum of the special schools turned out to be a laboratory wherein a new national curriculum was developed. Vogeli (1971) described the situation as follows:

> [These special] schools provided the opportunity for experimentation with curricular designs that would ultimately be introduced into the mathematics syllabuses of the entire secondary school system of general education. Topics from abstract algebra, probability and statistics, numerical analysis, topology and non-Euclidean geometry—which had been considered too difficult for the eighth through eleventh grades—were successfully introduced into the syllabuses for the special schools. The results of the experimentation supported the arguments of mathematicians and teachers who had been urging—in opposition to the official ministerial sanction of the polytechnic reforms—that the entire Soviet school mathematics syllabus be reorganized and modernized.
>
> The anti-polytechnic and pro-reformist impetus drew support from outside the educational policy-making community, that is, from the leadership of the industrial, technological sectors of the Soviet economy, who felt the need for workers, technicians and scientists with an increasingly sophisticated knowledge of mathematics.

The Soviet Union, like the United States, Japan, and Europe, is engaged in post-industrial technological development and expansion. The Soviet space program reflects a high level of achieved mathematical competence and demands continued effort in theoretical mathematics and applied technologies. If the former Soviet school curriculum and syllabus in mathematics was adequate to support the industrialization of the Soviet economy, it was evidently not considered adequate to support the ongoing needs of science and technological development.

Participation by Soviet mathematicians and mathematics educators in international meetings and conferences on mathematics education informed them of developments and achievements abroad. The inclusion in European and American school programs of abstract algebra, probability and statistics, numerical analysis, computer and applied mathematics, and other areas not represented in the Soviet ten-year school syllabus undoubtedly must have aroused the interest and concern of Soviet mathematicians. Although initial informal proposals for reform went unheeded, in part because of the persistence of efforts to devise a practicable polytechnic program, the success of the advanced, theoretical curricula in the special schools provided the support that the proponents of reform were seeking. (pp. 8-9)

As a result, the newest Soviet version of "new school mathematics" was developed, under the leadership of Academician A. N. Kolmogoroff, in the years from 1966 (when work on it began) until 1975 (when it was to be installed in all Soviet schools). Reliable reports from individual teachers seem to indicate that this latest Soviet curriculum has not met in an entirely satisfactory way the test of viability under the conditions of egalitarian implementation on a vast scale. Probably that was too much to expect. As usual, two questions remain, not yet satisfactorily dealt with: "For which students is a more sophisticated curriculum appropriate?" and "How can one acquire a cadre of teachers of the requisite (high!) level of technical subject matter competence, combined with the necessary pedagogical ability?"

The specific mathematical content of the 1975 curriculum is roughly as follows:

*Grades 1 to 3 (ages 7 to 10):*

In these grades are included the operations of addition, subtraction, multiplication, and division; the basic laws (commutative, associative, distributive, etc.) for these operations; the beginnings of algebra (on an introductory unpressured level); measurement (including area and volume); the basic geometric concepts of point, line,[1] and angle; and

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[1]Here, again, the need for specificity arises. Just what is a young child supposed to think that a "point" actually is? How does he deal with its idealized (unreal) nature--if at all? [cf. Davis, 1976c.]

elementary surveying,[2] actually carried on outdoors. Shabanowitz (1978) says of the curriculum for grades 1 through 3: "The connection between geometric, algebraic, and arithmetic material is an important feature of the mathematics program in the primary grades" (p. 56). The same is true for some, but not all, of the "new math" curricula in the U.S.

*Grades 4 and 5*

The arithmetical operations are extended to fractions and decimals.[3] Extensive use is made of number lines. Simple equations are used for physics ($s = v \cdot t$, etc.) and for geometry (areas, volumes, etc.). Plotting points in Cartesian co-ordinates is introduced, as are positive and negative numbers. Geometric topics include: use of ruler, protractor, and compasses in geometric constructions; the theorem that vertical angles are congruent; parallel lines; theorem on sum of angles of a triangle; congruence of triangles; Theorem of Pythagoras. (All proofs are described as "intuitive.")

*Grades 6 through 8 (ages 12 through 15)*

The concept of *function* is introduced in grade 6. Other topics include: inequalities; graphical representation of functions; the properties of integer exponents (without proofs); polynomial functions; and the solution of systems of two equations in two unknowns (both algebraically and graphically).

*Grade 7*

Topics include: the study of rational functions, including transformation of expressions, as in

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} = \frac{y - x}{y + x}$$

$$\frac{x^2 + ax + bx + ab}{x^2 - a^2} = \frac{x + b}{x - a}$$

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[2] Surveying in grades 4, 5 and 6 (U.S. style) was also a feature of some, but not all, of the various U.S. "new math" programs. Lauren Woodby, especially, developed teaching units for "outdoor mathematics."

[3] Once again, questions arise about specifics. Would not the outdoor experiences with surveying require fractions or decimals? The topic can hardly be new at the fourth-grade level. (Presumably what happens in grades 4 and 5 is a more thorough and systematic treatment.)

binary numerals (and decimal-binary conversions); absolute and relative errors, including the formulae

$$\Delta\left(\frac{a}{b}\right) = \frac{b \quad \Delta a - a \quad \Delta b}{b \quad (b + \Delta b)}$$

$$\frac{\Delta \ (a + b)}{a + b} = \frac{\Delta a}{a} - \frac{\Delta b}{b}$$

the function $y = \sqrt{x}$ ; and certain special functions such as the greatest integer function and the absolute value function. Geometric topics include: circles; chords, arcs, and central angles; intersecting and tangential circles; inscribed and circumscribed circles; and translation and rotation. There is also a separate course in descriptive geometry and engineering drawing.

*Grade 8*

Topics include: the solution of quadratic equations; Vieta's coefficient rules; arithmetic and geometric progressions; easy examples of proofs by mathematical induction; fractional exponents; informal treatment of irrational numbers; the exponential function; and the logarithmic function. This includes formulas such as

$$\log_B A = \log_B C \log_C A \ .$$

At this point the students are about 15 years old, and this seems to mark the end of one stage of mathematical study. Up until now this has been a program for *all* students. After this point has been reached, students' paths begin to diverge. The work of the "eight-year school" has been completed.

*Grades 9 and 10*

At this stage, for those students who go on, the *required* coursework in mathematics is reduced slightly (from 6 hours per week to 5), and *electives* make their appearance in a larger way. Geometric topics include: measurement results obtained from similarity principles; the trigonometric functions; law of sines; and law of cosines.

*Grade 9*

Topics include: limit of infinite sequences; irrationality of $\sqrt{2}$ ; limits of functions; continuity (defined by $\epsilon$, $\delta$ method); the derivative developed from the study of concrete examples involving velocity[4]; followed by undertaking the general study of

[4]For a similar program in the U.S., see Davis (1976b) and Davis, Jockusch, and McKnight (1978).

derivatives; vectors; trigonometry; differentiation of trigonometric functions; differentiation of exponential and logarithmic functions (and applications to problems of growth and decay in physics, biology, and economics); anti-differentiation; definite integrals; surface areas and volumes of solids of revolution; the harmonic oscillator; solution of certain second-order differential equations; combinatorics; probability; and Chebyshev's Theorem and the law of large numbers. Geometric topics include vector geometry and proofs of theorems in axiomatic Euclidean synthetic geometry.

*Optional Topics*

What has been described above, especially for grades 1 through 8, represents essentially the *required* curriculum for *all* Soviet children.[5] The difficult problem of providing for individual differences must now be faced. In addition to the special schools, the new (1975) Soviet curriculum tries to provide for individual differences by means of *optional* mathematics courses. These do not replace any of the basic courses, but are taken *in addition to them*. According to Shabanowitz (1978), main emphases in optional courses, which begin at grade 7, include: transformational geometry, complex numbers (including De Moivre's Theorem and Euler's Theorem), the solution of differential equations, non-Euclidean geometry, further study of the axiomatic method in geometry, vector spaces, computer programming, and numerical analysis.

As a suggestion of grade-level placement, geometric proofs using Cavalieri's theorem appear as an optional topic in grade 8 (14- and 15-year-old students); a more thorough study of the field of the reals and an introduction to complex numbers both appear in grade 9; while additional topics in the theory of probability, an introduction to computer science, and a course in programming digital computers all appear in grade 10 (16- and 17-year-old students).

In the effort to define the 1975 Soviet curriculum more precisely, it is helpful to look at typical questions from the examinations that are used at the end of the school year to determine which children will be promoted to the next grade.

*For promotion from grade 4 to grade 5:*

A state farm sent 11 tons 975 kg of fruit to a cannery, and 3 tons 950 kg less than that to various stores. Apples accounted for $\frac{3}{5}$ of the weight of all fruit shipped.

[5]A considerably more detailed discussion of the 1975 mathematics curriculum is presented in Shabanowitz (1978).

How many boxes were needed to pack the apples, if one box can hold 30 kg of apples? (Shabanowitz, 1978, p. 73)

*For promotion from grade 6 to grade 7:*

Solve the following equation and check it:

$$(3.2 - 2x)\ 2.5x + x(5x - 0.5) = 1.5.$$ (Shabanowitz, 1978, p. 74)

*For promotion from grade 9 to grade 10:*

Find the value of r such that the graph of the function $y = x^r$ will pass through the point

$$P\left(\frac{3}{4}, \sqrt[4]{\frac{108}{4}}\right)$$

(Shabanowitz, 1978, p. 75)

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# Chapter V

## One View of the Value of Studying Mathematics Education Research and Development in the Soviet Union

Robert B. Davis

### A. *What Do We Need?*

What a supermarket has to offer to you depends not only on what it has in stock, but also--and usually more importantly--upon what you need. So, before we consider Soviet R and D, we should perhaps look around the U.S. and try to decide what we need. This is the hardest part of the task. Even if we could all agree on what is available in Soviet R and D literature (and we are hardly well-enough informed to be able to do that), there are surely markedly different views as to what is lacking here in the U.S. That, of course, is the reason for providing four independent chapters by different authors in this report. We begin, then, with one observer's shopping list. As we identify items, we shall consider what might be available from a study of Soviet work.

I ask the reader to bear in mind that what follows is a personal list, written in "objective" language only for the sake of simplicity.

#### 1. *Description*

There is no more grievous lack in typical U.S. research and development work than the absence of enough *descriptions*, and good enough ones. A very common kind of research report will assert something like this: "The use of art to motivate arithmetic in grade 5 was found to yield no significant difference in student gain scores, as measured in grade equivalents by CTBS tests."

But, typically, the report does NOT tell you how "art" was "used" to relate to arithmetic. Did students use wax crayons to color in purple-outlined pictures that had been run off on a ditto machine? Did they make original "sculptures," using wood and metal scraps from the junior high school shops? Did they make silk screen prints?

How was the art *related* to the mathematics? Was art actively used as a *reward*--after getting a score of 90 or better on an arithmetic test, a student could go to the art room for a two-hour session? Or were the art and the mathematics inextricably interwoven, perhaps in the task of building a massive mobile that required an *a priori* determination of balance points, using the computation of turning moments? If so, was the specific project of such a nature that computations were in fact inherently necessary, or were they a gratuitous requirement imposed by the teacher on a task that could be done more simply by "cut-and-try" non-mathematical methods?

How about the teachers? Did a single teacher deal with both art and mathematics? If so, was the teacher fully competent in, and enthusiastic about, both art and mathematics separately? Was the teacher enthusiastic about the combination?

How good were the specific tasks that were used? What were they, specifically? Did they involve spatial visualization, as Marion Walter's "milk carton cutting" does?

*The common omission of such details suggests a prevailing view that questions (and answers!) of that type are not important. But every good teacher knows that this is false. What you do is important, but the details of how you do it are no less decisive in shaping the outcome.*

A decade or two ago, U.S. research and development efforts seemed to argue that such "details" were neither necessary nor appropriate to *abstract science*, to the "scientific" study of education. This was clearly false: very often the details are of decisive importance.

More recently a different defense has been used: *The study of education needs to concern itself with policy questions.* A superintendent of schools, or a principal, can make decisions and set guidelines only at an abstract level, such as scheduling art and mathematics teachers to work together; the superintendent cannot be involved in the details of program implementation. This argument is no less specious than its predecessor: why should research and development be aimed primarily (or solely) at administrators? The needs of teachers--and, in fact, of parents--are just as great, and they make different kinds of decisions, where the importance of detail is undeniable.

Far more research and development is needed where *exactly* what is done is recorded and reported in the fullest possible detail, and where the entire outcome is similarly recorded and reported. This can sometimes be done by detailed verbal descriptions, but can sometimes be handled better via videotape, audiotape, or film.

Education, after all, is not merely "applied science"--indeed, it is not mainly applied science. It is also very largely an art, and must be treated as one. The long-term history of music shows unmistakable periods of progress. But these have depended upon knowledge of the details of performance, usually achieved by hearing live performances. Could Beethoven have composed as he did if he had never *heard* a performance of the music of Mozart, Haydn, Bach, or any other predecessor, but had seen merely "abstract" descriptors, written not in musical notation, but in ordinary German prose: "The composition begins with a loud chord, followed by a silence, then a rising sequence of notes, in which the violin is prominent..."? Clearly not! The details of composition and performance are critical. These are captured in live performances, in recordings, and in sheet music--but not in abstract verbal descriptions.

To some extent an alternative approach--fundamentally different--has been provided by Piaget. He has observed the details of certain behavior, and has described it quite well.[1] His impact in the United States has been useful, but some Soviet studies offer more, and some Soviet discussions of methodology are especially valuable. Consider, for example, this passage by Krutetskii (1976):

> As an illustration of the aforementioned deficiency of test studies, we can cite the research of J. Murray, devoted to an analysis of geometrical ability and published in 1949. The author, using a testing technique that completely ignores the process of solving test problems, found a correlation between success in mastering geometry and particular abilities: reasoning ability, the ability to "grasp" spatial relations, and so on. The same kind of research is represented by the more recent investigations of mathematically gifted adolescents by W. A. Kennedy and others, the research of E. Locke on the correlation between indexes of gifted science students' intellectual abilities and personality traits obtained in the classroom and their out-of-class achievement, and many more. It is hard to understand how theory or practice can be enriched by, for instance, the research of Kennedy, who computed, for 130 mathematically gifted adolescents, their scores on different kinds of tests and studied the correlation between them, finding that in some cases it was significant and in others not. The process of solution did not interest the investigator. But what rich material could be provided by a study of the process of mathematical thinking in 130 mathematically able adolescents! (p. 14)

Krutetskii does not leave us in doubt as to the kind of alternative he has in mind; he gives us examples--and these examples show a good level of detail in observing and describing a student's actual thought processes:

> Let us consider a few examples to convey as clearly as possible the essence of our criticism. In most cases where the same test result has been obtained, the mental processes that have led to the result can be essentially different. And this very difference can be the most valuable material for judging an examinee's psychological traits--his abilities. With this in mind, we shall discuss a few examples from our work: examples of different psychological means of arriving at the same result.

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[1]Yet Piaget's alternative, though very important, has not entirely filled the need for alternative R and D methodologies for the U.S. We return to this theme below in a later section of this chapter.

1. *Problem*. Three friends visit the library on different days: the first once every 3 days, the second once every 4 days, and the third once every 5 days. The last time they were at the library together was on a Tuesday. In how many days will they again be at the library together, and what day of the week will it be?

*Solutions*. Pupil G.S. (seventh-grade boy) rapidly wrote down a series of consecutive numbers starting with 1 and began quickly crossing out numbers: every third one (with a line), every fourth (with a dot), and every fifth (with a cross). He got the right answer mechanically: 60 days. He rapidly counted off the days of the week and got Saturday. Correct answer; solution time: 2 min., 2 sec.

Pupil Yu. A. (seventh-grade girl) thought a bit, then said, "So this will be the least common multiple!" Without hurrying, she calculated ($3 \cdot 4 \cdot 5 = 60$). She divided 60 by 7, obtained 8 weeks, with a remainder of 4 days. She declared: "Wednesday, Thursday, Friday, Saturday. Two months from Saturday." Correct answer; solution time: 1 min., 22 sec.

For both the result is the same. The psychometrician evaluates both pupils as equal (at best, making a "correction" for the solution time). The *processes* of solution are at entirely different levels.

2. *Problem*. Write down 100, 101, 102,...What do the three dots mean? What is the first number you have written down? What is the first digit? The third digit? *Question:* What will the thirteenth digit be? The twenty-first digit?

*Solutions*. Pupil A.K. (fifth-grade boy) simply continues the series and primitively counts off the specified digit. The answer is correct; time: 29 sec.

Pupil S.B. (sixth-grade boy) reads aloud (and shows on his fingers) the digits from 1, until he reaches the needed result. Correct answer; time: 21 sec.

Pupil R.V. (sixth-grade girl), having noticed after some reflection that each number has three digits, reads aloud in groups of three, without reproducing the whole series. Correct answer; time: 24 sec.

For all three, the result is the same: the problem is solved correctly. Moreover, the solution times approximately coincide. Any psychometrician is obliged to rate all three pupils as altogether equal. But it is clear even to someone inexperienced in psychology how different were the psychological paths leading to the result.

To show how important a pupil's unique problem-solving methods can be and how much an analysis of the process can contribute to research, we cite an example of the solution of two problems by three pupils (we shall give brief descriptions of the solutions). The test problems were taken from Werdelin's study. Pupils in the Moscow schools did the solving.

*Problem 1.* If we add 360 to a certain number, we obtain the same result as we would if we multiplied this unknown number by 4. What is the number?

*Problem 2.* A mother is three times as old as her daughter. Ten years from now she will be only twice as old as her daughter. What is the mother's age?

*Solution by Sasha R. (seventh grade).* Quickly, without pausing, composes equations and solves.

| | | | | |
|---|---|---|---|---|
| 1. | $360 + x = x \cdot 4$ | $360 = 3x$ | $x = 120$ | |
| 2. $x$; | $x + 10$ | $3x + 10 = 2(x + 10)$ | | $x = 10$ |
| $3x$; | $3x + 10$ | $3x + 10 = 2x + 20$ | | |

*Solution by Raya Ts. (seventh grade).* Quickly, without pausing, draws a diagram:

```
           + 360
1.  □ =  □ □□ □  □ = 120
2.  □ +  ○
    □ □ □ + ○
    □ □ □○ = □○ □○   □ = 10 years (daughter)
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*Solution by Robert N. (seventh grade).* Does not write or draw anything. Quickly says, for (1), "Add 360 and take it four times—it's all the same. Then 360 is three equal factors. The number is 120." (2) "The difference between mother and daughter will always make up two initial ages of the daughter, and in 10 years these two initial ages will equal to the next age of the daughter, that is, in 10 years the daughter will be twice as old. The daughter is 10 and the mother 30 years old."

All three pupils solved both problems; all three were considered able pupils; the solution time for all three was approximately identical. A psychometrician would have to rate all three as entirely equal. But even a superficial analysis of the process should suggest essential differences in these pupils' mental processes.

We have dwelt on this matter because it is important to show concretely the different possibilities that are missed in

testing and the erroneousness of its conclusions. The psychometrician, in each instance cited above, must conclude that the pupils are equal in their manifestations of mathematical ability, whereas analysis of the process suggests that we are dealing with different levels of mathematical ability in the first two problems and with different varieties of mathematical ability in the second two. (Krutetskii, 1976, pp. 14-16)

....

As a result of all that has been said above, one must agree with Teplov, who wrote that testing on the whole has not cleared the way for a study of individual psychological differences: "In a scientific respect it has proved unavailing, and in the sphere of practical application sometimes downright injurious". (Krutetskii, 1976, p. 17)

The omission of detailed *descriptions* in typical U.S. educational research is, in part, a misreading of the history of science. In recent years, one or another kind of *experiment* has been important in the traditional sciences (though not, as Kuhn (1962) points out, nearly as important as the popular folk-lore suggests). However, in medicine, physics, and chemistry this age of experimentation had been preceded by many *centuries* of careful observation and description (cf., e.g., Davis, Jockusch, and McKnight, 1978). *The lack of comparable descriptions in education must be overcome; a major effort at describing treatment, describing outcomes, and describing thought processes is needed. The use of audiotapes, videotapes, and film can greatly facilitate this process. And, as the Krutetskii excerpts should suggest, this is a matter where we CAN get some help from Soviet work, especially in the description of alternative "surface" thought processes.*

How far must one go in *describing, observing,* and *recording?* My answer is: very far, indeed! We do not at present possess the abstract descriptors necessary to enable us to determine the essential nature of learning experiences, of thought processes, or of outcomes. Consequently, we must sometimes go even this far: record the learning experiences on videotape; record sample outcome behaviors on videotape; and report that 'for the 361 students studied (for whom an incomplete set of descriptors exists and would be used in describing them), educational experiences that looked like *this* produced, in at least 83 percent of the cases, an outcome that looked like *that.*'

In music, the "outcome" tapes might record a student recital, or in the case of mathematics, they might show a student as he or she is confronted with a problem of a new unfamiliar type, and allow us to watch and to listen as the student tackles the problem. Sometimes it is more effective to have several students work together on the unfamiliar new problem, since conversation between the students will sometimes help to reveal more clearly the way the students are thinking about the problem.

A (small) part of this matter is sometimes dealt with, within the U.S., as the distinction between *process* and *product*. To some observers (especially laymen), it seems obvious that product--the "bottom line," as it were--is all-important, whereas process is incidental and usually insignificant. Of course this is false. One question, intended to emphasize the fallacy in "counting right answers" as a measure of achievement or understanding, is this: Would you rather have a watch that (without resetting) is correct twice a day, or would you prefer one that (again, without resetting) is correct once a year? If one "counts the number of right answers," the first watch gets (at least) 730 correct answers in 365 days, and should presumably be chosen in preference to a watch that gets only one "right answer" in this period of time. But of course a watch that doesn't run at all will be correct twice every day; a watch that is right only once a year must, in fact, gain (or lose) 3.9 minutes each day, and is perfectly satisfactory for most ordinary purposes.

Exactly similar phenomena happen with students. A student whose procedure for, say, subtracting several-digit numbers contains one minor flaw may, as a result, get all--or nearly all--of the answers wrong. Yet the task of identifying and correcting this "one minor flaw" may be quite simple, after which the student may get all of the answers right.

Another student may get substantially more answers correct, but such may be the nature of the flaws in that student's procedures (or the difficulties of some other nature that he or she experiences), that significant remediation may be essentially impossible. For serious remediation--or even for correct assessments in general--one MUST deal with "processes," rather than "products." Krutetskii and many other Soviet researchers hold to precisely this point of view.

## 2. *Interview Studies*

A corollary to the importance of *description* is the need for *interview studies*. These have played a large role in Soviet work, as one sees immediately from, for example, the work of Krutetskii (1976) cited earlier. Among U.S. researchers, see Erlwanger (1975), Alderman, Swinton, and Braswell (1979), and Burton and Brown (1978). Other Soviet researchers include Kabanova-Meller (1970), Talyzina (1970), Yakimanskaya (1970), Zykova (1969), Yaroshchuk (1969), and Doblaev (1969).

## 3. *A "Structural" or "Metaphoric" Theory of Human Information Processing*

Chemistry could not have achieved its modern successes without the development of a "structural" or "metaphoric" theory in terms of atoms, molecules, atomic nuclei, orbital electrons arranged in layers, etc. Had chemistry attempted to deal only in "real-valued variables" such as density, pH, atomic weight, and so on, it could not have made long-term significant progress (cf., e.g., Davis and McKnight, 1979).

In a similar way, educational studies must relate to a "structural" or "metaphoric" theory of human information processing. The appropriate *kind* of theory, perhaps not yet correct in detail, but surely moving in that direction, is being built by M. Minsky, S. Papert, Marilyn Matz, John Seely Brown, R. Schank, and others (cf., e.g., Minsky and Papert, 1972; Matz, 1979; Brown, 1979; Davis, 1977). This is extremely encouraging, but this type of "metaphoric" theory is still of interest only to a small group of researchers. A larger discussion of such matters should be beneficial. This might be more likely if some Soviet conceptualizations were included in the discussion. The work of Tikhomirov and Terekhov, and of N.V. Rivkus, reviewed by Rachlin in Chapter VII, would be examples of relevant Soviet studies.

There are said to be many recent Soviet studies that follow the kind of "artificial intelligence" paradigm that we have in mind. These studies are presently unknown in the United States, but persistent inquiries could presumably locate them.

### 4. *Family and Home Environment Studies*

There is no greater need in the study of U.S. education than the need for studies of the educational effect of different patterns of parental behavior, home environments, and other determiners of student self-concept, student preferences, student values, student habits, student aspiration levels, and so on. It is overwhelmingly apparent that students differ, one from another, and that these differences are often very stable, not easily modified by schools (cf., e.g., Davis and Douglas, 1976). These differences are not well-described by I.Q. scores--they relate to regular habits of doing assigned homework on time, to working carefully, to being curious and inquisitive, to persistence in working on difficult or tedious tasks, to self-confidence, etc.

What kinds of parental behavior influence the development of good work habits, curiosity, and self-confidence in children? Virtually nothing--beyond folk-lore--seems to be known about such matters. Yet nothing is more fundamental to the outcomes of education.

Soviet educators have long stressed the importance of the "upbringing" of children--indeed, there are special professionals known as "upbringers." This distinctive Soviet notion--involving responsible citizenship, good work habits, respect for teachers, etc.--bears the Russian name "vospitaniyeh" [ВОСПИТАНИЕ]. Thus, there can be special interest in studying the development of good work habits and similar factors in Russian children, and--since the Soviets devote much attention to such matters--one can hope that there are many existing Soviet studies worthy of translation into English and study by U.S. researchers. Where typical Russian practice diverges sharply from typical U.S. practice, there is always a possibility of useful studies. This divergence clearly occurs in the matter of the very close supervision of children in Russia, quite unlike American practice (cf. Smith, 1977; Bronfenbrenner, 1970).

There is the further possibility of new studies by U.S. scholars in the Soviet Union, by Soviet scholars in the U.S., or jointly in the two countries. One important study of this type already exists: Bronfenbrenner's *Two Worlds of Childhood: U.S. and U.S.S.R. (1970)*. Given the importance of family studies, much more might be undertaken.

Before continuing our shopping list of what is needed in R and D, it may be well to pause and ask what *goals* we have for the entire venture.

### B. *Goals for the Improvement of Mathematics Education*

Again with the admonition that this is a personal list, I propose these goals for the overall program of research and development in education:

1) Creating programs such that most students learn more mathematics than they do at present;

2) Making it possible for most students to derive more gratification from the study of mathematics than they do at present;

3) Making it easier for students to relate mathematics to their personal career choices [this includes both a "curriculum" aspect--trying to make the curriculum relevant to these career choices--and also a "student" aspect--making students less fearful of mathematics (and better prepared in mathematics), and hence more willing (and able) to select careers that involve mathematics];

4) Increasing student *understanding* of the mathematics they do learn;

5) Increasing competence in "routine" mathematics, such as solving systems of linear equations, etc.;

6) Increasing competence in heuristic analysis and the various aspects of creative and original work in mathematics;

7) Giving students a better idea of the various major aspects of mathematics (e.g., where is it useful, where is it less likely to be useful, etc.);

8) Matching mathematical training to the economic needs of society;

9) Accelerating the pace of education so that young people are not forced to remain in student status any longer than necessary;

10) Getting a deeper understanding of the processes of learning mathematics and solving problems involving mathematics.

### C. Further Consideration of What Is Needed

We resume our "shopping list" from Section A:

#### 5. Complexity

The study of learning in the United States has focused mainly--almost exclusively, it sometimes seems--on simple cases, often too simple to shed much light on the learning of mathematics or the acquisition of skill in problem solving. The following quotation from Kilpatrick and Wirszup (1969) puts the case clearly:

> In view of recent concern about the place of axiomatics and proof in the American school mathematics curriculum, it is surprising that there have been virtually no research studies in this country concerned with pupils' thought processes in making a geometric proof. In fact, the classic work on proofs in geometry is still Fawcett's study, published in 1938, which deals only with promoting the ability to prove theorems and not with describing or analyzing the process. Needless to say, if American mathematics educators have shied away from investigating processes of geometric proof, American psychologists have completely avoided the topic, with its complex tasks, its potential for uncontrolled sources of variation, and its lack of observable behavior. Soviet psychologists, however, have not been so reluctant. As the articles in this volume indicate, there has been substantial interest in the Soviet Union in how pupils prove geometric theorems, which is seen as part of the larger question of how they solve problems in geometry. In contrast to Polya's distinction between "problems to find" (with unknown, data, and condition as their principal parts) and "problems to prove" (with hypothesis and conclusion as their principal parts), Soviet investigators appear to view all problems as composed of a condition (containing the data) and a requirement (or unknown). Pupils are apparently taught that when faced with a geometry problem they should identify the principal parts, locate them in the diagram given with the problem, and attempt to recall relevant concepts and theorems. In other words, they are taught to use suggestions much like those given by Polya, although the approach is more formal and the terminology different. (p. 1)

#### 6. Laboratory Schools

In some ways one of the most troublesome criticisms of educational research is that it makes little contact with reality, that it is "merely college professors writing for other professors."[2] One observer recently remarked that the whole adventure of the "new math" seemed to have been

[2]For one interesting assessment, see Rising (1978).

a storm on the surface of the ocean, having no perceptible effect on the actual teachers, classrooms, and students, who of course dwell far below on the ocean floor.

If chemists had no laboratories, chemistry might be an abstract study, lacking contact with reality. If inflation, recessions, monopolies, and trade deficits did not occur, economics could conceivably be an abstract study. What could save educational research from the fate of losing contact with reality? The author--who, for several years, has been privileged to teach in a high school that is, in fact, a laboratory school for the University of Illinois, and who thus has first-hand experience in the discipline that is enforced by the reality of teaching pre-college students--would argue that educational research and development requires *laboratories*, and that for ventures of significant size (NOT in terms of large numbers of students, but rather in terms of major innovations, and following students for three or more years), these laboratories need to be *laboratory schools*.

If one adds to this the unfortunate separation between LAS (Liberal Arts and Sciences) Mathematics Departments and Colleges (or Departments) of Education, one has a clear recommendation: the typical U.S. university should own and operate a pre-college school, not one associated only with the Department of Education, but rather one where the LAS mathematics Department is responsible for mathematics, the LAS Chemistry Department is responsible for chemistry, and so on.

Exactly this pattern *can* be observed in a few Soviet "special schools" operated by major universities, one of which has Kolmogoroff (one of the world's greatest mathematicians) as principal. Study of this Soviet practice might help Americans to see the value of such arrangements. Vogeli (1971) reported that a small number of "special schools," created and operated on this principle, have come to have a profound impact on all Soviet education.

7. *Curriculum Revision*

The U.S. and the U.S.S.R. have tried remarkably similar "new math" curricula, and for generally similar reasons. Although the U.S. venture has been one of the most interesting developments in American education, it has received essentially no serious study (except, perhaps, for one evaluative study by Dilworth, 1971). But subjects deemed unworthy or unsuitable for study in a single nation sometimes acquire greater appeal when two nations can be involved.

In any case, continuing revisions in the school mathematics curriculum will remain necessary for both nations, and the Soviet practice of extensive published discussion could be helpful also to the United States. *There needs to be far more written public discussion of the school mathematics curriculum.*

## 8. *The Role of the Teacher: Partly Pre-programmed, Partly Spontaneous*

Is the teacher to be thought of as the airline stewardess, who reads a prepared statement from a plastic card, or as the pilot who improvises travelogues over the public address system? Is the teacher an actor or actress, presenting well-rehearsed lines scripted by someone else, or is the teacher an improviser, responding in a new and unique way to a new and unique challenge?

Every position, from one extreme to the other, has its advocates and its critics. At least one approach used successfully in the U.S. uses some carefully scripted episodes, combined with some teacher decision making in response to student initiatives or other transient considerations. To continue this "dual" approach, and to make the most of it, more attention needs to be paid to perfecting the scripted portions, and to helping teachers find increasingly powerful ways of dealing with the "spontaneous" portions.

Something can be learned from Soviet practice. With all teachers in the nation using the same textbook, it becomes possible for the Soviets to give great attention to the "scripted" part of the teacher's work.

As an illustration of the relation between the "scripted" and "spontaneous" components in teaching, the Madison Project developed the "pebbles-in-the-bag" activity to introduce negative integers. This was a carefully crafted and carefully tested activity, intended to be taught with careful fidelity to the "script" (Davis, 1967a, p. 54); for this activity, extensive teacher training was provided. But the Project also assumed a large component of flexible or spontaneous teacher behavior; for this component, a different sort of teacher education was provided.

One can hardly imagine a U.S. curriculum that is uniform, nationwide, in the Soviet fashion. Nonetheless, the opportunity that is thereby created for a large investment in teacher education is very attractive. One has only to read Bantova (1971), Taruntaeva (1971), Pchelko (1971), or Shepetov (1971) to see how great attention to detail is possible when one is dealing with a fully-scripted part of the teacher's job. These authors are able to cite individual problems in the textbook *by number*, and can discuss exactly how problem number 623 (say) is to be handled. Even the *title* of the Shepetov essay is indicative of the specificity of *a priori* determination in the Soviet curriculum: "Certain Features of the New Mathematics Textbook for the Fourth Grade."

These essays have the appearance of being genuinely helpful to teachers, and for that part of the teacher's job that is fully scripted, such help is clearly appropriate, and even fairly easy to provide.

But the Soviets provide help also for the flexible or spontaneous part of the teacher's job. Consider, for example, the following:

> A striving to avoid a presentation of material where everything begins with formal definitions and is developed according to the laws of logic is clearly expressed in the textbook. In many cases, a practical problem serves as grounds for the introduction of new subject material, and the problem helps give an idea of the importance of the new knowledge. The solution of such a problem serves as the basis for subsequent generalizations. There are a fair number of examples of this; consider only the points devoted to the laws of arithmetic operations. The teacher should not only support this methodological approach by using such problems for developing problem situations, but should show initiative, and this is even more true when there are many occasions for such investigations. For example, the textbook begins the presentation of the concept of percent in the traditional way--i.e., from determining a percent--and there is not sufficient motivation for introducing this concept. Creative forces are also necessary in achieving the other object of problem teaching: the organization of independent student work controlled by the teacher and directed toward opening up new relationships and laws in the text material. (Shepetov, 1971, p. 127)

> One should shorten considerably the number of problems solved with written questions. In acquainting students with a new type of problem, it is sufficient to solve three or four problems with a written formulation and resort later to written questions in cases where students have difficulties in their oral formulation. Then one need not regret time spent to solve the same problem by different methods, to work out a given problem and solve inverse problems, to compare problems reducing to the same relationship, and, after solving similar problems, to form a generalization and make up a formula for its solution. In a word, one should not regret time spent on anything that is associated with independent investigations of the pupils and on methods for solving a given problem. (Pchelko, 1970, p. 121)

The Soviet system of providing *detailed help* for teachers is extremely important. A suggestion of how some of the advantages of this method could be achieved in the U.S. is given in the following section.

## 9. *Alternative Projects for A Unified Approach to Curriculum Improvement and Teacher Education*

To save time, let it be agreed from the outset that a single, uniform, nation-wide curriculum would be unacceptable in the U.S. (although the existing extent of diversity is not very great and often virtually indetectable). One could *increase* the amount of

diversity and alternatives—a desirable thing to do, as the genetics of biologically successful species demonstrates—and at the same time establish within the U.S. some of the Soviet-style advantages of helping teachers in highly specific ways. Indeed, the U.S. was well on its way to achieving this a few years ago. The mechanism was as follows (we use the Madison Project as an illustration, but the essence of the argument is that a *multiplicity* of such projects are needed; too few projects would lead to an unacceptable monopoly):

a) In trial teaching, carefully "scripted" units were developed, refined, and tested.

b) Procedures for handling the "flexible" or "spontaneous" or "responsive" part of teaching were worked out, again using direct trials with students *as part of the design process itself* (and NOT merely for testing *after* design work was done).

c) Typical classroom lessons were recorded on film or videotape, so that other teachers could see for themselves *exactly what* was done, and *exactly what the resulting outcome was*.

d) A large teacher education program was set up, in cooperation with the public school systems in Chicago, San Diego County, New York City, Los Angeles, Philadelphia, and elsewhere, that reached over 30,000 teachers.

This program was achieving significant successes, but for various reasons it declined, both in size and in vigor, and no longer exists as a large, coordinated program.[3,4] While it did operate, however, it addressed many of the problems discussed in this chapter: by developing materials directly in classroom teaching situations, and NOT in separate writing sessions, it maintained effective contact with the reality "at the bottom of the ocean"—i.e., where the students are; by recording typical lessons and typical student performance on film and videotape, it preserved a great deal of depth, specificity, and (as Jerome Bruner pointed out) "immitatability"; it addressed both the "flexible" and the "scripted" parts of the teacher's job; and, in its large-scale teacher training program, *because teachers WERE offering a common set of learning experiences*, it enjoyed the advantage of a *specificity* and a *concreteness* not entirely unlike Soviet practice.

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[3]However, smaller versions of the program do continue, with undiminished vigor, as in the case of the special summer program offered by Wesleyan University in Middletown, Connecticut, in cooperation with the Association of Mathematics Teachers in New England.

[4]Some of the reasons for the decline were intrinsic to the program itself. One should not, however, overlook the influence of external influences which were entirely unrelated to the program itself (cf. U.S. House of Representatives, Reports, 1975, 1976).

What could make this approach acceptable to U.S. colleagues would be *having a multiplicity of such projects*. (Indeed, there did exist such a multiplicity: Beberman's UICSM Project operated in this way, as did Loman's USMES Project, among others.) This approach addressed many aspects of the general problem of improving and updating school mathematics. It may still represent one of the best hopes for future progress.

The similarities and differences of U.S. and Soviet experience in approaching the task of revising the school mathematics curriculum can be a source of extremely valuable information and insights. The U.S. experience by itself has not been adequately studied--but, again, it may prove easier to study two nations together, rather than one by itself.

## 10. *The Explication of Alternative Belief Systems*

For some teachers and some parents, "education" is a process of *telling* things to students, and somehow causing the students to commit the things to memory. The "things" told may be "facts," or they may be rote procedures (such as the long division algorithm).

For other teachers and other parents, "education" is something entirely different. Consider, for example, the assertions of the Soviet psychologist S. L. Rubinshteyn, paraphrased by Julius Goldberg as follows:

> .... during the transition from an act's connection with practical thinking to its association with theoretical thought, a re-orientation occurs during the systematic teaching of sciences to school children. That is, practical activity is an extremely important stimulus for the formation of thought. ...The task confronting the educator is not only to communicate knowledge to his students, but [to] teach them to think. (Goldberg, 1978, p. 373)

The point is that there are many different conceptualizations of "learning," with different techniques for producing learning and different criteria for determining when learning has occurred, quite comparable to the situation of "electricity" in Benjamin Franklin's day, as reported by Kuhn (1962). Is the presence of "electricity" to be detected by *attraction*, or by *repulsion*, or by sparks, or by magnetic effects, or by some combination of these? Is "learning" to be detected by the student's ability to repeat back what he or she has been told, or by the ability to relate new situations to certain fundamental ideas, or by the ability to use heuristics to cope with entirely new problems, or by some combination of these?

Much of the controversy over the "return to basics" in U.S. schools is in fact a confrontation between conflicting conceptualizations of "learning" and "education." Unfortunately, far too

little has been done to identify and describe--"explicate" is a good name for the process--the various alternative conceptualizations of learning that are prominent in the U.S. today. But, as Bruner has remarked, "the fish will be the last to discover water," and it is probably easier to recognize various U.S. conceptualizations while we are engaged in the task of identifying Soviet conceptualizations.

Moreover, Soviet scholars seem more inclined to try to identify and compare alternative conceptualizations. We have seen something of Rubinshteyn's position. Consider, now, the following:

> In the Soviet psychology of teaching and learning mathematics one can distinguish two basic trends or schools of thought. The first of these was created by N. A. Menchinskaya, P. A. Shevarev and their students; the second was established by P. Ya. Galperin, N. F. Talyzina, D. B. Elkonin, V. V. Davydov and their followers. In the Soviet Union, S. L. Rubinshteyn was a pioneer in formulating the theory that activity is the determining factor in the functioning and development of thought. The difference between Menchinskaya and the Galperin schools consists in the different means each selects for concretizing the unity of psychology and activity. (Goldberg, 1978, pp. 374-5)

To a distant observer, it seems that there is strong disagreement between the Menchinskaya group and the Galperin school. Consider the following:

> At the present time, the approach to the learning process in the U.S.S.R. is characterized by three types of learning. The following classification was suggested by Galperin, himself:
>
> *THE FIRST TYPE.* The mastering of knowledge occurs spontaneously, and is an unguided process. The formation of concepts and skills in acts proceeds according to trial and error. Prescribed conditions are insufficiently distinguished. Correct acts are formed gradually by virtue of selection, which is controlled by a final result. The transfer to new tasks is almost totally lacking. It should be noted that this type is, in fact, the one which was developed by the Menchinskaya school.
>
> *THE SECOND TYPE.* A student works under continuous supervision, receiving all the necessary instructions which will result in a correct action or an action with only minor errors. If new tasks have something in common with the tasks used during instruction, then a transfer of acts to the new tasks takes place. This type was developed by Galperin and his followers.
>
> *THE THIRD TYPE.* Advantages are added to those of the second type of learning. Acts of the student, formed without trial and error, occur under those conditions during which he not only is compelled to carry out precise

instructions and demands and must submit to them, but also when he understands why these tasks are caused. In this type of learning the cognitive interests and abilities of the students are broadly developed, and a generalized transfer of skills and abilities to new knowledge is obtained. This type of learning was proposed by Galperin as the perfection of his second type. It is being developed by both Menchinskaya's and Galperin's schools, the latter apparently achieving more significant results. (Goldberg, 1978, p. 379)

11. *Some Points of Contact Between Soviet Studies and U.S. Mathematics Education*

In this section we look, very briefly, at six specific points of contact between Soviet studies and U.S. mathematics education. Another cautionary note is called for: this is being written at a quite considerable distance from the Soviet investigators, with the inevitable possibility of substantial misunderstandings and misconceptions.

a) *"Discovery" teaching*

This has, of course, been discussed within the U.S. (Shulman and Keislar, 1966; Davis, 1967b), but mainly in an inconclusive way. Sooner or later, further and more incisive discussion is needed.

One finds points of contact in Soviet writing, as for example in the work of Rubinshteyn ["psychological processes not only appear, but are formed, during activity" (Goldberg, 1978, p. 373)]. Such points appear perhaps especially in Rubinshteyn's careful description of one aspect of one version of "discovery" teaching:

> ... For example, if a student cannot solve a problem without outside assistance, he should not be given a prepared solution, but rather should be given the separate stages of analyses which, after employed, enable him to proceed. This technique should be used by a teacher who wants to teach his students how to think, and not just solve typical problems using a mass of prepared patterns. (Rubinshteyn, as reported in Goldberg, 1978, p. 374)

b) *The growth of cognitive structure in a student's head (the "jig-saw puzzle theory"*--cf. Davis, 1972)

Rubinshteyn, again, states:

> A child can master and use knowledge presented to him only if the inner conditions for their assimilation ~~and use~~ have been well established in his thinking. (Goldberg, 1978, p. 374)

c) *Interview studies*

The case for using interview studies to map out a student's thinking as he or she solves a mathematical problem is beginning to become clearer in the U.S.; it has long been accepted in the U.S.S.R.:

> A new method of psychological research was created. Observation, conversation, and finally real experimentation came to be widely used...[around 1940]... (Goldberg, 1978, p. 374)

Even earlier than Rubinshteyn (who wrote in 1940), we of course have work of Vygotsky in 1926:

> Vygotsky felt that it was essential to study the acts of thinking in response to an instructional process. With his groups of associates, he researched the development and mastery of scientific concepts by children as [that development] occurs during the learning process in school. This work demonstrated the importance of a gradual creation of concept systems during instruction. (Goldberg, 1978, p. 371)

d) *The teaching experiment*

Within the U.S., optimal methods of using teaching situations for the design and development of new learning experiences or teaching units are still matters of disagreement and uncertainty. There is an obvious value in learning more about the Soviet "teaching experiments."

e) *Cognitive processes in algebra and geometry*

The Menchinskaya conceptualization of information processing involved in thinking about polynomials (Goldberg, 1978, p. 375) can be related to a similar, but different, conceptualization presented in Davis, Jockusch, and McKnight (1978).

Even more clearly, cf. the Galperin conceptualization:

> ... The subject's activity serves as the initial basis for the formation of mental acts. The formation of mental acts begins with preliminary acquaintance with the demands and conditions which ought to answer the acts. The act, then, takes place with external objects, as in the case where a calculation is made with the assistance of concrete objects. Here, the act proceeds under the control of the things themselves, and the content of the future mental act is formed. Further, verbalization occurs, for example, calculation aloud. Action with objects passes, i.e., is transformed,

> into actions with concepts. Finally the action evolves into a mental plan, acquiring the traits of the inner cognitive process. (Goldberg, 1978, p. 377)

This bears an evident relation to the development of "integrated procedures" and "meta-language," as described in Davis, Jockusch, and McKnight (1978) and Davis and McKnight (1979), including the Newell Conjecture.

f) *Uses of mathematics*

The importance of learning mathematics in a context of *using* it, as developed by such persons as Edith Biggs, David Page, and Seymour Papert, is a major theme within the U.S. An apparently similar position among Soviet psychologists has been stated by Galperin:

> Galperin and his followers maintain that without the application of knowledge, its mastery is impossible. Application of knowledge is the basic means of mastering [the knowledge], not the concluding stage. There is no knowledge until it is applied. Knowledge is acquired in the process of the student's activity. (Goldberg, 1978, p. 378)

Galperin's statements are very close to the statements of Biggs, Papert, and Marion Walter. Continuing:

> The problem of instruction consists not in teaching how to apply abstract information in practice, but in showing how abstract knowledge is built on concrete content...Through practical and mental acts the student discovers the essence of the phenomena and masters the associated concepts. (Goldberg, 1978, pp. 378-9)

To be sure, there are some major unanswered questions in the area. In using Dienes' MAB blocks to teach place-value numerals, one "goes from the concrete to the abstract" and Galperin's description seems entirely applicable, but this becomes less clear in more complicated cases. There is the further question of whether a sizeable multiplicity of different "concrete embodiments" is preferable to one main concrete embodiment.

## 12. *A Third Methodology*

The traditional U.S. methodology for educational research has been, as we have argued above, a relatively remote methodology, relying on questionnaires, multiple-choice tests, statistical analysis, and an emphasis on "product" rather than "process." The limitations of this method are coming increasingly to be recognized (cf., e.g., Ginsburg, 1972), as the remarks in Krutetskii (1976) make especially clear.

Piaget's "clinical interview" methodology has provided a welcome and important alternative of a radically different type, relying on a very small number of children (sometimes only three), not at all randomly selected, on the presumption that when profound matters are clearly observed and described in the case of a very few children, there will be relevance to all children--a relevance guaranteed, not by sampling procedures or sample size, but rather by the fundamental nature of the phenomena themselves. Piaget has provided evidence on this; his main results are indeed universal.

While it is in no way Piaget's fault, the "Piagetian alternative" that has evolved in the U.S. has developed its own liabilities. For one thing, the fundamentals studied by Piaget are relatively remote from typical "school-type" tasks, such as the long division algorithm or the solution of word problems. But more seriously, as Papert has pointed out, the implications of Piaget's results have tended to be reversed in American analysis. Whereas Piaget in fact identified fundamental cognitive abilities that children develop, *even in the absence of formal instruction*, and which provide a foundation for school learning, the popular U.S. interpretation reverses this, and contends that Piaget found limitations on a child's "readiness," barriers to what a child is ready to learn. The Soviets reject this re-interpretation of Piaget, and argue--as, for example, Davydov does --that young children are in fact able to learn far more mathematics than was previously thought possible, and more *abstract* mathematics at that.

But beyond discussions pro and con, *Soviet practice in using school-related tasks in task-based interviews demonstrates a third possible methodology that is a valuable addition to the typical U.S. abstract/statistical methodology and the typical Piagetian methodology.*

## 13. *The Ability of Children*

One of the most stable discoveries of the past twenty years has been the repeated demonstrations, by many different researchers, that very large numbers of children can learn considerably more mathematics than used to be thought possible. This has been demonstrated in California by William Johntz (1975), in Florida by Burt Kaufman, in Maryland by Julian Stanley (1977), in Massachusetts by David Page, nation-wide by the C.E.E.B. Advanced Placement program, and elsewhere by other investigators (cf. David, Jockusch, and McKnight, 1978, pp. 259-269). In the Soviet Union exactly the same discovery is reported by Davydov,[5] by Vogeli (1971), and by others, and is

[5]"D. B. Elkonin and V. V. Davydov organized a broad, psychological-pedagogical experiment of several years duration which involved many teachers and students. The basic conclusions of this work led to a fundamental re-examination of the mathematics curriculum which took place in the lower grades. As a result of the investigation, it was concluded that younger school children are more advanced in the areas of abstract and theoretical thought than had been previously realized, and a new mathematics curriculum was designed on the basis of these investigations." (Goldberg, 1978, p. 388)

indicated by the new Soviet textbooks (cf., e.g., Kolmogoroff et al., 1975), "special schools," and elective mathematics courses.

*But the larger question is unanswered: we have not yet found effective ways to build on this usually undeveloped ability of students in ways that can benefit society and the individuals themselves.*

14. *Design*

String quartets must be *composed;* plays and novels must be *written;* automobiles and airplanes must be *designed.* Effective learning experiences must also be *designed*--but in education, design is the dog that never barked. While great attention is devoted to *evaluation,* virtually no effort within education is devoted to *design.* Here, again, what has been insignificant--or nonexistent--in one country may become important when two countries are to be considered. (Besides, the Soviets write and talk much more about questions of *educational design*--and when you are designing a nation-wide program for all of the U.S.S.R., perhaps you had better!)

15. *Refuting the "Linear" Assumption*

In recent years one has occasionally heard, in the U.S., the idea that there is a "standard" sequence for research and development: first one does research, then one refines and revises the results of the research, then these results become the basis for "development" work, then *these* products are evaluated, then revised, then re-evaluated, and finally disseminated.

This theory is contrary to history.

In fact, there is *no* "standard" sequence. Perhaps more often than not one *starts* with development and implementation--with routine practice and daily work. This has been the case with beer, glass, pain-killers, metallurgy, thermodynamics, wine-making, navigation, the building of the great cathedrals, Roman plumbing, astrology/astronomy, surgery, acupuncture, and the curing of leather, to name just a few familiar examples.

In this matter, it appears that Soviet practice in educational research has been closer to the common successful patterns of the history of science and technology, starting (as they often do) with *observation, interviews, description,* and their well-known (but, in the U.S., not clearly understood) "teaching experiments." (Goldberg attributes much of this to the influence of Rubinshteyn in the early part of the twentieth century.)

Adherence to the "linear" pattern in the United States inevitably constitutes a pressure toward abstract, academic research, toward lack of adequate contact with reality, toward neglect of

that which is subtle, profound, elusive, or a matter of *art*--in a phrase, toward "college professors writing for other college professors," neglecting the practitioners' expertise of teachers, and ignoring the children whose classrooms and lives remain untouched "on the ocean floor." It is important to develop a recognition of where research really starts, which can be *almost anywhere* that people deal thoughtfully with important questions.

## 16. *Refuting the Misuse of Tests*

Within the U.S. it is commonly believed that objective tests are the best source of "hard data." This is clearly *not* the case, as anyone can see if they will look directly at schools and children. Recently, there have begun to appear some well-reasoned attacks on the typical assumptions and typical practices of "testing," "measuring," and "evaluating." Particularly worthy of attention are Banesh Hoffman's *The Tyranny of Testing* (1962); "The Score Against I.Q." and other articles by Morrison and Zacharias, Whitla and Whitla, Schwartz, Taylor, and Perrone in *The Myth of Measurability* (Houts, 1977). It is, of course, hard to improve much on Krutetskii's extremely accurate, and extremely negative, analysis, some of which is quoted elsewhere in these chapters (Krutetskii, 1976).

The present use of tests in the United States does *not* have a scientific basis [Morrison (1977) discusses this with his usual combination of scientific sophistication and expository skill], it *does* have many harmful consequences and implications, and it *is* based on an essential circularity. Alternatives (as Perrone points out) do exist or can be created.

The moment that one looks *carefully* at individual test items, or at typical conditions of test administration, or at the underlying philosophical justifications, or (as Morrison points out) at the important contributions made by innumerable valuable people who "test badly" ("make low scores on the tests"), it becomes apparent that *tests are NOT giving us meaningful "hard data" at all.* They are giving us *numbers* with all the simplicity and apparent precision that numbers imply--at least emotionally. But numbers are not automatically descriptive of much of anything, however much they may *seem* to be.

Much could be gained if the role of tests in the United States could be appropriately circumscribed. They *can* do some things, but far less than they are usually presumed to be capable of doing. Here is a situation--as Krutetskii makes clear--where Soviet commentary and Soviet practice can help a great deal.

## 17. *Is School Mathematics Really Mathematics?*

Krutetskii pursues an interesting discussion: is school mathematics a rote process, preliminary to the study of "real" (creative)

mathematics, or is school mathematics a true part of real, creative mathematics?

> Before defining the basic concepts with which we shall be concerned, we must note that mathematical ability can find expression at quite different levels of activity. We shall treat the concept of mathematical ability in two aspects:
> 1. As creative (scientific) ability--ability in scientific mathematical activity, which yields new results or achievements that are significant for humanity, a product that is valuable in social terms.
> 2. As school ability--ability in the study (learning, mastery) of mathematics (in this case, the school mathematics course), in the rapid and successful mastery of appropriate information and skills.
>
> Both psychologists and mathematics usually distinguish these two levels.
>
> The question arises: Can abilities for mastering mathematics be regarded (and under what conditions) at least to some extent as a manifestation of genuine mathematical ability, in the proper sense of the word (scientifically creative)? Or are they such different categories that when the former are manifested it is absolutely impossible to say anything about the potential future appearance of the latter? When we study abilities for mastering mathematics, can we believe that to some extent, within certain limits, we are studying mathematical ability properly speaking?
>
> Some psychologists (especially foreign ones) are inclined to make a sharp distinction between these two levels of mathematical ability, and thus if one looks at achievement at the first level one can conclude nothing with confidence about achievement at the second level...
>
> To us, the difference between the two levels of activity does not seem absolute. In studying schoolchildren's mathematical abilities, we have in mind not simply their educability. Our study will indeed concern the pupils' school abilities, but *creative* school abilities, related to an independent creative mastery of mathematics under the conditions of school instruction, to the independent formulation of solving these problems, to the invention of proofs of theorems, to the independent deduction of formulas, and to finding original methods of solving non-standard problems. All of this undoubtedly is also a manifestation of mathematical creativity. If the presence of a creative principle is a criterion for mathematical thinking proper, we must not forget that mathematical creativity can be not only objective but subjective. Establishing specific criteria to differentiate the creative thought process from the uncreative, A. Newell, J. C. Shaw, and H. A. Simon note the following indications of creative thinking: (1) the product of

mental activity has novelty and value both subjectively and objectively; (2) the thinking process is also novel in that it requires a transformation of previously accepted ideas or a denial of them; (3) the thinking process is characterized by the presence of strong motivation and stability, observable either over a considerable period of time or with great intensity.

If we proceed from these criteria (which we believe to be valid), then many mathematically gifted children demonstrate a kind of creative mathematical thought. Anticipating somewhat, we might point out that some of the gifted pupils we studied literally "discovered" for themselves individual sections of the school course in algebra and geometry. They were discovering what had long been well known. The product of their creativity had no objective value, but for the pupils themselves (subjectively), it was doubtless a discovery, an invention, an independent achievement of something new. In a certain sense, this activity most certainly is part of mathematical creativity. The rediscovery of what was known can be creativity, and subjectively the product can be new and original--as Newell, Shaw, and Simon emphasize. R. Carpenter makes the same observation: that the product can be objectively uncreative but the process still be creative.

Thus we can affirm that there is a definite connection between the two levels of mathematical ability. Hadamard asserts that there is only a difference in degree or level between the work of a pupil who is trying to solve algebra and geometry problems and the work of a discoverer in mathematics--both kinds of work are similar in nature. Kovalev and Myasishchev discuss the idea that these two levels "are joined by a genetic bond. Under favorable conditions of formation, the abilities of a pupil-mathematician become the abilities of a real scientist." Yu A. Samarin shares this point of view.

Thus, we believe that the question whether abilities for learning mathematics can be regarded as a manifestation of mathematical ability in the proper sense of the word should be answered affirmatively. A thorough, independent, and creative study of mathematics is a prerequisite for developing an ability for creative mathematical activity--for the independent formulation and solution of problems that have new socially significant content. Precisely for this reason an investigation of schoolchildren's mathematical abilities is the first step on the way to a study of mathematical ability in its higher manifestations. (Krutetskii, 1976, pp. 66, 68-69)

The two positions being argued here represent, in fact, two essentially different conceptualizations of mathematics education. The distinction is an important one, and needs further discussion in the United States--hopefully in ways that will reach teachers, parents, and students. Entirely different programs of school mathematics will be designed, depending upon which view one takes. If pre-college mathematics is a rote precursor of "real" mathematics, then the old paraphernalia of flash-cards, memorization, and drill--perhaps modernized with the aid of computers--becomes appropriate. But if pre-college mathematics is a genuine part of "real" mathematics, if students need *creative* experiences in mathematics from an early age, then programs such as David Page's and the Madison Project become desirable or even necessary.

## 18. *Distinctive Soviet Practices: The "Collective"*

Obviously, an opportunity for study exists wherever Soviet practices are substantially different from those in the United States. Particularly important among these is the use of "the collective"--essentially, collective responsibility for the performance of an individual. Thus, a definite group of students is responsible for the behavior and achievement of each individual student who is a member of this group. This is a very conspicuous feature of Soviet education.

U.S. practice has tended in an opposite direction, toward more complete individualization, often with each student working alone. A growing number of U.S. teachers are beginning to complain that something is lost in the process of individualization--there is no longer pride in group membership, no longer the feeling that "one has to hold up one's end" as a "member of the team." This may well be an important phenomenon that has hardly been studied at all. Many U.S. teachers and many U.S. schools do NOT make use of "being on the team" in *academic* matters, although the majority probably do in relation to football, basketball, and other athletic activities. However, for that minority of schools and teachers who do try to use pride in "being on the team" as a source of achievement motivation in academic work, the trend toward individualized study has usually been seen as a potentially serious loss. (A particularly promising program developed by Layman Allen, and demonstrated in the Detroit schools, has not attracted the attention that it deserves.)

The whole area of group cooperation and group responsibility deserves more serious study. This cannot be undertaken lightly--one can have pride in "being a Marine" (to cite a familiar U.S. example) *only because this IS seen as a matter for pride in the relevant circles.* In short, for group responsibility and team performance to be effective, *one must really believe in it.* It is probably NOT useful to try to pretend.

19. *Pioneer Palaces*

Another distinctive Soviet practice is the combining of what, in the U.S., might appear as Boy Scouts, Girl Scouts, museums, zoos, the YMCA and YWCA, the Little League, Hebrew lessons, private piano and ballet lessons, after-school clubs in journalism or computers, the Future Farmers of America, and so on, into a single institution, the *Pioneer Palace*. In the U.S.S.R. this is, in effect, a second *kind* of "school system," entirely separate from the regular academic school. From, say, 8 a.m. to 3 p.m., Soviet children attend the regular academic school, where they study the Russian language, Russian literature, mathematics, English, physics, chemistry, Russian history, world history, geography, biology, etc. This is in fact a highly academic school. From about 3 p.m. to perhaps 6 p.m., Russian children attend the *Pioneer Palace*, where they study ballet; serve as zoo keepers for their own animal collection; study interior decoration and cooking; have a model railroad club, a model airplane club, and a computer club; have a club that corresponds in English with children in other nations; and so on.

The Pioneer Palaces are important for several reasons. As *independent* institutions, NOT part of the regular school system, they provide what in effect amounts to the greater control that is possible when you have two independent variables instead of one. While, for example, the Soviets have chosen to move slowly on the introduction of computers into the regular school program, because--since there is a single nation-wide program for *all* schools in the entire U.S.S.R.--the vast size of the venture demands slow and careful innovation, it has been possible to move very quickly to introduce computer clubs into Pioneer Palaces. Moreover, the training of staff at Pioneer Palaces is different, so that one is not locked into a single ponderous system of teacher education, teacher certification, etc. In effect, the Soviets have the advantage of *competition*--there are two separate, and potentially competing, school systems, not invisibly linked by a single system of teacher education and certification.

Some of these advantages could be achieved in the U.S.; for example, by introducing "computer science" clubs and programs into museums, Boy Scouts, Girl Scouts, and other agencies. Some very successful beginnings in this direction already exist.

20. *Soviet Examinations*

No general consideration of Soviet mathematics education would be complete without looking, at least briefly, at the surprising phenomenon of Soviet written examinations. *By U.S. standards, Soviet examinations are remarkably--almost unbelievably--difficult.*

For admission to Moscow State University, to study physics, students had to demonstrate their ability on a test that (in 1969) included these questions (which are fairly typical of Soviet exams):

1. Solve the inequality

$$\log_5 x + \log_x \frac{x}{3} < \frac{(\log_5 x)\quad(2 - \log_3 x)}{\log_3 x}$$

2. Two teams of checker players A and B meet. According to the contest rules, each participant of one team plays one game with each participant of the other team. The total number of games to be played is 4 times greater than the number of all players on both teams. Due to illness, however, two players (one on each team) could not appear in the match, with the result that the number of all games played in the match was 17 less than intended. How many players entered the match for team A if it is known that there were less players in team A than in team B?

3. Solve the equation

$$(\sin x)\,(\cos\frac{x}{4} - 2\sin x) + \cos x\,(1 + \sin\frac{x}{4} - 2\cos x) - 0.$$

4. Two circles tangent to one another are inscribed in angle $\alpha$. Determine the ratio of the radius of the smaller circle to the radius of a third circle which is tangent to the first two and one of the sides of the angle.

5. A certain quantity of brick must be moved at a construction site. At the disposal of the builders there are three forklift trucks with load capacities of 100 kg. for the first, 200 kg. for the second, and 300 kg. for the third. A quarter of all the brick was carried in nine loads on one of the forklift trucks. Then 10 more trips were made on another truck. After this, one-third of the initial quantity of brick remained to be carried. After all the brick had been moved, it turned out that in the entire moving process the third truck made two times as many trips as the second. In the moves the trucks were loaded to full capacity. How many trips did each of the three forklift trucks make?

6. Solve the inequality

$$2\cdot x^{\log\frac{1}{2}x} - x^{-\log\frac{1}{2}x} < -1.$$

7. An airplane makes a landing and moves uniformly along the ground for a certain time with the velocity $v$ m./sec. Then the pilot applies the brakes and the motion of the aircraft becomes uniformly decelerating such that in each second the velocity reduces by 2 m./sec. The distance from the point of

landing to the point of complete stop is equal to 4 km. The ratio of the time in which the aircraft covers the first 400 m. to the time in which the aircraft covers the entire distance along the ground is equal to $\frac{4}{65}$. Determine the velocity $\underline{v}$.

21. *A Faster Pace*

The pace of mathematics instruction in U.S. schools is, typically, almost unbelievably slow. We have observed a 45-minute lesson, taught to above-average students, that consisted *only* of problems of the type

$$x^2 \cdot x^3 = \qquad [\text{Ans.: } x^5]$$

$$p^{10} \cdot p^2 = \qquad [\text{Ans.: } p^{12}]$$

and so on. It turned out that this was the *second* of two lessons taught to this class in this form.

Now, for some conceptualizations of what mathematics learning really is, this snail's pace makes sense: if learning mathematics is nothing but the creation of simple stimulus-response ("SR") bonds, and if one *always* wants the stimulus $x^2 \cdot x^3$

to evoke the response $x^5$,

then (to some extent, at least) the practice makes sense. But this conceptualization is defective.

For one thing, the range of relevant stimuli is very large. It includes

$$(x^2 + 1)^2 \; (x^2 + 1)^3$$

and

$$2 \log \sin x + 3 \log \sin x$$

among many others. (Menchinskaya deals with this: cf. Goldberg, 1978, p. 275.) But further, as Bruner points out, one of the goals of education is to produce an educated person who is not bound to a unique, machine-like response to a given situation, but who has a broad enough repertoire of possible behaviors to be able to produce a variety of responses, and who has access to sufficiently fundamental knowledge so that his or her responses can have an authoritative correctness in addition to an (at least occasional) surprising novelty. This clearly applies to architecture, and is a major part of Beethoven's special genius in musical composition--*but it also applies to mathematics.*

Kazuko Suzuki, when 13 years old, knew the Archimedean Postulate in the form:

If $a$ and $b$ are any positive numbers,

then there exists a positive integer

N such that

$aN > b$.

She was asked to prove that, for any $\epsilon > 0$, there exists a positive integer N such that $\frac{1}{n} > \epsilon$ provided $n > N$. She solved this problem with apparent ease--*but notice that some surprising changes of form are required in order to accomplish this.* This is NOT a case where single, "automatic" responses are called for. *You must be prepared to try something original and highly imaginative* (cf. Davis, Jockusch, and McKnight, 1978, p. 90). Kazuko attended a special school, not typical of U.S. high schools. In a more typical school she would not have encountered a task of this subtlety.

The point, however, is that two 45-minute periods are far too much time to devote to problems of the single type $x^2 \cdot x^3 =$ . The pace of mathematical instruction in Soviet schools seems to be more rapid. This should be of real interest to educators in the United States.

22. *Residential Schools*

The Soviets operate *residential high schools* for mathematically gifted students (Vogeli, 1968). This is a more serious provision for mathematically gifted students than any that presently operates in the United States. This calls, at the very least, for study--and probably for some reflective thought.

A particularly important aspect of these schools is that they are owned and operated by liberal arts university mathematics departments. We return to this point below.

23. *High Schools for Computer Scientists*

Vogeli (1968) describes special Soviet high schools for computer scientists. Vogeli also discusses parental concern that these schools would shanghai students into immediate employment in low-level jobs, when these students would be better off to continue a longer period of study, leading to higher level employment in the broad area of computer science.

The need to try to reduce the number of years of study required for various professions is clear, as the Newman report pointed out a decade ago. Some personal experience is relevant here. The author works for the PLATO computer project at the University of Illinois (actually "CERL," for Computer-Based Education Research Laboratory). This is probably the largest computer system in the world devoted entirely to helping students learn (and NOT to keeping office records,

etc.). A great deal of valuable computer programming at CERL is done by students in junior and senior high school, some of them as young as 12 years old. Much of this is very complex, high-caliber, *professional* work. A division of CERL personnel into stronger and weaker programmers would find quite a few adults in the "weaker" category, and quite a few high school students near the top in the "stronger" category.

There is, then, some realistic possibility of professionally useful employment of teenagers in jobs that should pay reasonably well (although at present they often pay nothing at all, because of child labor laws). At the very least, some sorts of "apprenticeship" programs could be created. Without denying the depth of study that can and should be undertaken by many students, it does not seem desirable to devote the first third, or thereabouts, of a person's life to study, perhaps through a Ph.D. or post-doctoral study, *before* the person begins to earn money from any significant professional work. Alternative arrangements *can* be created, and are needed. (In some few individual cases, this happens already, as in the case of the students who work at CERL.)

24. *Correspondence Schools*

At the pre-college level, the Soviets also operate some *correspondence schools* and some *part-time evening schools*. These provide more advanced studies in mathematics and/or computers. Similar arrangements--or alternative arrangements to serve the same goal--could be valuable in the United States, and not just in mathematics. How can we offer courses in Russian, in Japanese, or in Mandarin to high school students in the U.S.? Not as part of regular school programs, in most cases--but perhaps as special summer programs, again operating on a residential basis for six or eight weeks? Still other arrangements are possible. (In Maryland, Julian Stanley has made imaginative use of special arrangements to provide programs for mathematically gifted high school students.) For relevant Soviet work, see Vogeli (1968), Vasil'ev et al. (1971), and Glagoleva and Gutenmakher (1971).

25. *Programs for Gifted Students*

Comparison of Soviet and U.S. programs for gifted students, *and specifically for mathematically gifted students*, leads to the unmistakable conclusion that the U.S. is failing to provide *serious* programs for mathematically gifted young people. What is done in the U.S. has, in most cases, the clear appearance of tokenism--something is done, but it is too little or too haphazard or too lackadaisical to make any real difference in the lives of the students (cf., e.g., Suppes, 1977).

26. *The Social Meaning of "Discovery Teaching"*

In the U.S. during the 1960s, when there was a lively interest in "discovery teaching" and "discovery learning," some pro-discovery teachers felt that one essential aspect was that *each student participated in group discussions during which SOME students discovered certain patterns, facts, or methods.* This group discussion was itself an important social reality. At the beginning of a typical session, a problem would be posed, and it would usually be apparent to everyone that no student in the group knew how to solve the problem,--or even how to approach it. But the discussion would move on. Possible lines of attack would be suggested, and would be analyzed, criticized, modified, perhaps rejected. Gradually, however, a suitable means of dealing with the problem would take shape. In one such lesson, recorded on film, the key "break-through" ideas were contributed by a boy named Lex and a girl named Debbie. Not every student made the key "discoveries"--*but every student was present and lived through the experience of being in the group when Lex and Debbie made the key discoveries, and they all recognized what had happened.*

This, some teachers felt, must inevitably affect the way *all* of the students perceive mathematics--it is something that you can think about. Appropriate methods *can be discovered*--because, after all, every student *had seen Lex and Debbie make the discoveries.*

This raises a question: is this the kind of thing that Vygotsky had in mind, when he said that "all specifically mediated human mental processes arise only in the course of social activity, in the process of cooperation and social intercourse"?[10] Given the very considerable hazards of Russian-English translation, it is not at all easy to arrive at an answer to this question.

Nonetheless, the *general impression* that one gets from much of the Soviet literature is a view similar to U.S. curriculum projects of the 1960's, and to U.S. cognitive studies in the 1970's: mathematics is complex and creative; subtle aspects are important; actual thought processes are important, and differ from one child to another (although there *are* important underlying patterns); "discovery learning," under appropriate conditions, is highly desirable; an elaborate pattern of ideas must be built up in a child's head, and only the child can build it; it is the teacher's job to help the child to build up this elaborate structure of interrelated ideas, and to help the child correct the structure whenever it is found to be in error.

This has a very familiar, and very welcome, sound to many mathematics education specialists in the United States . . ., but, once again, given the difficulties of translation from Russian to English, and given our sparse sample of source materials, it may all be a pleasant illusion, a comfortable misunderstanding, and nothing more. We shall need to study far more of Soviet research, and to have available more good translations, before we can decide.

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[10] Cf. Chapter VI of the present report.

27. *Who Should Do R and D?*

The history of education in the U.S. shows the gradual creation of a separate institution, with a separate cadre of professionals, to provide for the pre-college education of young people. The degree of separation has, over the past hundred years or so, become extreme (cf., e.g., Cremin, 1961). It is often believed that this very degree of separateness in itself constitutes a liability, and considerable evidence can be found to support this view. But how can the separateness be overcome? Many who now stand outside of education must become involved in it, and in genuinely significant ways. An American cannot help but be impressed by Soviet universities--and mathematics departments, not schools of education--operating correspondence schools, part-time schools, or residential schools for gifted students. Or by Kolmogoroff serving as principal of one of these special high schools, or being listed as one of the authors of pre-college textbooks.

The Pioneer Palaces also form a link between the education of young people and the larger society, with scientists and administrators taking time off from their jobs to contribute directly to pre-college education (Johntz's Project S.E.E.D. has also arranged this, on occasion, within the U.S.).

A decade or two ago the demands of the computer industry, the demands of the vast space effort, and the expansion of higher education within the U.S. pulled mathematically talented people irresistably away from work with pre-college students. Employment of mathematically trained people is now lagging, and hence an opportunity exists in the U.S., also, to get many more mathematically skilled people involved directly in pre-college education. But, of course, some sort of national movement to accomplish this might speed the process considerably. In the 1950's and 1960's, the National Science Foundation took the lead in this effort. Who might do it in the 1980's?

28. *Neurological Studies*

As Romberg reports in Chapter VI, Soviet education makes somewhat greater use of neurological studies than is common in the United States. The work by Martha Denkler, M.D., and Patricia Davidson, Ph.D., at Boston Children's Hospital is strikingly similar to Soviet work, and could serve as a point of contact for closer international cooperation in this very new (and potentially very important) field.[11]

29. *The History of Soviet Schools*

In the United States we necessarily become so occupied in the day-to-day problems of our schools that we tend to lose sight of the

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[11]Papers by Denkler and Davidson are listed in the references for Chapter IX.

social and economic problems that our schools—or possible alternative institutions--need to be addressing. The creation of the strong system of community colleges (which overlap high schools in the content they teach, such as trigonometry), under the leadership of John Brademas and others, may be one of the most important developments of recent years. *Headstart*, *Follow-Through*, *Upward Bound*, and the *Job Corps* were other promising interventions, although their contribution to solving social and economic problems is probably less clear.

There is a broader historical, social, and economic pattern. At the present time, in 1979, it is still the case that many adult Americans are functionally illiterate, and many are extremely deficient in saleable skills. No educational programs seem to be addressing this problem effectively.

There may also be less obvious patterns and pressures. Where are technological or economic changes creating new educational needs? When Americans left farms in large numbers to migrate to cities, we were not prepared to deal with the phenomenon, and it contributed to the development of urban slums and the decay of urban schools. When the automobile and improved highways led to a migration of the affluent out of cities, we were unprepared for the erosion of urban tax bases and the loss of able students. What is happening now that none of us is noticing, and for which we will find that we were unprepared?

The study of the history of Russian and Soviet schools can be very eye-opening: the consideration of the adjustments the Soviets have made, and their reasons for making them, cannot fail to give Americans a broader perspective on social/economic problems in general. It is not so much that either country has been entirely successful in arranging education so as to eliminate social and economic problems--rather, the point is that we can learn from the various efforts. And we can get more ideas from the failures than we can from the successes. The point is NOT to imitate, but to reconsider (cf., especially, Vogeli, 1971).

30. *Methodology in General*

The improvement of education, in many different respects, is a definite need that should be addressed in a serious way. The present array of methods for observing, describing, and analyzing education seems not to be adequate. Far too often one gets reports of "no significant difference" in cases where knowledgeable observers suspect that there *was* a difference, but for one reason or another it was not observed and described by the specific methods employed, perhaps because these methods were not sensitive enough, or because they were not focused on the places where the difference made itself felt, or because several effects canceled each other out (a specific treatment, for example, may be better for some educational goals, or for some students, and worse for other goals or other students).

What is clearly called for is a greater determination to observe, describe, and analyze more effectively--and, at least at first, that means by employing a far larger array of methods. The superb journalism of the *Wall Street Journal* is regularly employed to describe and analyze complex human undertakings--it deserves to be tried out seriously in education. Readers can easily think of other serious methodologies. U.S. educational R and D has been too exclusively committed to psychometrics, questionnaires, statistics, and low-inference observational protocols. Studying the Soviet literature inevitably raises questions about a wider array of methodologies.

31. *Demographics: Education and Careers*

A major task of education is to equip everyone to contribute significantly to the economic well-being of society. There are many unanswered questions in this area. Quite a few of these have not even been seriously discussed.

*Shanghaiing.* We borrow this word from Harold Howe, who once suggested that *normal schools* were created "to shanghai young women into becoming teachers." What we have in mind, however, is not primarily teaching. In most fields there are higher-level jobs and lower-level jobs. In medical services there are medical specialists, "general practitioners" or "family doctors" or "primary care specialists"; there are nurse practitioners, graduate nurses, registered nurses, practical nurses, nurses aides, and paraprofessionals. Who is to play which role? Who is to earn $200,000 a year, and who is to earn $10,000 a year?

In the days when genetic determination of performance capabilities was a finely developed theory, things seemed to sort themselves out nicely. Everyone could be presumed to rise to his or her highest level. The theory didn't really work, but for a time it survived, and the sons of doctors became doctors.

Our present U.S. theory seems remarkably egalitarian--nearly everyone seems to be considered a potential candidate for nearly anything, and the main problem is often seen as achieving a fair distribution of races, sexes, and religions in every line of work. This theory represents a correct recognition of certain aspects of reality, but it omits others.

By "shanghaiing" we mean offering young people who aren't sure of career directions fairly quick access to the lower levels of various careers, but at the price of blocking them out of subsequent advancement. We make it easy to become a para-professional; but virtually impossible for a para-professional to become a surgeon, and--*this is the key point*--we entice young people into becoming para-professionals *without giving them fair warning, in a form they can assimilate, as to what this will entail in future decades in their own lives.* The delicate point is a kind of "truth in labelling" question. Just what is society's responsibility to young people, to warn them of what their future may hold, *and to do so in a form that the young people will really understand?*

Of course, presumably *somebody* has to work at low level jobs, as dishwashers, filing clerks, dental hygienists. Who? The women's movement in the U.S. exacerbates the problem--it was formerly felt that easy access to low-level jobs was appropriate for women, but not for men, on the grounds that a woman would work at the level only briefly or sporadically, being mainly occupied as a wife and a mother. Hence it could be argued that a fair bargain was in effect--women invested less time, money, and energy in their education, and were granted quicker access to income-producing jobs; this fairly well matched up with what it was presumed that they wanted. From men, more income would ultimately be expected, so a larger initial investment--as in completing college and medical school and internships--could reasonably be expected.

That theory is becoming increasingly untenable--but what else is in sight to replace it? The "career ladder" discussions in the *Washington Monthly* are a partial response, not as yet taken seriously by the general public.

*Time.* As another aspect of the demographics problem, we have observed earlier that children as young as 12 years old may be able to produce quality work in, for example, computer programming. This *kind* of activity has at least two features:

(i) it deserves to earn money, but usually doesn't, partly because of child labor laws (and probably also because nearly everyone assumes that "kids should be grateful for the opportunity to learn"--which of course, may actually be true);

(ii) it differs from delivering papers and waiting on tables in that it can have direct and important relevance to the student's education and future professional careers.

More provision could be made for this kind of experience, and possibly ought to be.

## 32. *Specific Cognitive Theories*

Within the United States, a general, not-yet-well-formulated kind of theory is beginning to take shape, concerning the human thought processes that are involved in learning arithmetic, algebra, and geometry (Burton and Brown, 1978; Matz, 1979; Brown, 1979; Davis, Jockusch, and McKnight, 1978; and other work, notably by Herbert Simon, Ira Goldstein, Seymour Papert, Allan Kaye, Marvin Minsky, Edwina Michener [Rissland], Jill Larkin, and others).

As discussed earlier, one has the *feeling* that this sort of work is rather in the spirit of much Soviet work. Yet the translations available to us do not seem to answer this question clearly. Is there, in fact, the prospect that this kind of American approach does have a Soviet parallel, and, if so, are the two lines tending to converge toward a single, more fully developed theory?

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# Chapter VI

## One View of Soviet Research in Mathematical Education

Thomas A. Romberg

The view presented in this chapter was primarily gained as a result of a two-week visit to the Soviet Union in December 1978. The purpose of the visit was to participate in a "Seminar on Teaching and Learning" with seven other U.S. educational researchers and several Soviet researchers at the Academy of Pedagogical Sciences in Moscow.[1] Twenty papers (seven U.S. and thirteen Soviet) were written prior to the seminar, translated into the other language, and made available to the participants. Three and a half days were spent intensively interchanging ideas from the papers with our Soviet colleagues. This was followed by trips to Tallin and Leningrad where we visited with scholars in institutions in those sites. Also, we visited schools in each city getting first-hand impressions of the impact of their research on teaching practices.

The titles of the papers prepared by the American delegation were:

T. A. Romberg, "The Implication of Curricular Issues to Research on Teaching and Learning"

B. J. Biddle, "Ideology, Social Planning, and Research on Teaching in the United States"

A. A. Bellack, "Contrasting Approaches to Research on Teaching

L. S. Shulman, "Recent Development in the Study of Teaching"

B. R. Tabachnick, "Teacher Education as a Set of Dynamic Events"

T. S. Popkewitz, "Educational Reform: Centagonistic Meanings and Institutional Life"

G. G. Wehlage, "Can Teachers be More Reflective About Their Work? A Commentary on Some Research About Teachers"[2]

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[1]The seminar was sponsored by the Soviet Ministry of Education. The American delegation was funded by the Council for the International Exchange of Scholars.

In summary, this set of papers emphasized the "teaching" theme of the conference. Each paper addressed issues about the most effective and promising trends in which research on teaching can be advanced. Also, there was explicit attention to ways in which ideology determines parameters of research in the United States.

The titles of the papers prepared by the Soviet participants were:

V. V. Krajevsky, "The Didactic Principles Underlying Teaching in Soviet Schools"

I. Ja. Lerner, "Curriculum as a Component of Education - Upbringing Process at School"

L. Ja. Zorina, "The Didactic Principles of Formation of the Pupils' Theoretic Reasoning"

N. A. Menchinskaja, "Problems of Self-directing of Cognitive Activity and Development of Pupils' Personality"

V. V. Davidov, "Basic Problems and Trends of Soviet Research in Psychology in Learning"

A. K. Markova, "Psychological Conditions of Motive Formation in the Process of Learning"

Z. K. Kalmykova, "Ways of Developing Pupils' Productive Thinking"

I. D. Zverev, "Education Content in Soviet School"

A. M. Pishkalo, "Features of Primary Education"

G. G. Maslova, "Some Aspects of Mathematical Education Reform in Soviet School"

V. G. Razumaovsky, "An Instructional Procedure Analysis and Ways of Improving the School Physics Course"

P. R. Atutov, "The Polytechnical Principle in Teaching the Fundamentals of Science"

O. F. Kabardin, "Optional Courses"[3]

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[2]The other member of the American Delegation was Beatrice Beach Szekely, editor of the English-language journal <u>Soviet Education</u>.

[3]The order of titles for both sets of papers are given here in the order of presentation at the seminar. An edited set of these papers will be published by Praeger later this year.

In summary, the Soviet papers emphasized "learning" rather than "teaching." Each paper was descriptive of particular aspects of the single, national school of pedagogy that exists in the U.S.S.R. The Marxist-Leninist ideology of materialism is often cited as the foundation of Soviet education theory, and was officially accepted as the fundamental context from which all descriptions emanated.

In spite of the difference in orientation of the papers--U.S. studies on teaching and Soviet studies on learning--the subsequent discussions between both sets of participants proved to be very fruitful. My comments in this paper were formulated as a result of those discussions.

I should emphasize that my comments are impressions, not facts. I am confident that the basic ideas expressed in this paper have some validity, but given the small amount of time, my understanding of the ways in which Soviets operate is limited. Their system could only truly be understood through a longer and more intense interchange. This is particularly true since discussions had to be carried out in different languages. Translations cannot capture the nuances of each other's ideological assumptions. We often use the same words with substantially different meanings.

To organize my impressions let me pose three answers to the following question:

What can be learned by examining the educational practices of a different culture like that of the U.S.S.R.?

*Answer 1*

My first answer is that *comparative studies make what one takes for granted problematic.* As long as one operates within a culture and its prevalent ideology, one takes for granted a host of features of schooling, mathematics instruction, and research. Some features portray only surface differences, such as "school operates five days a week" (Soviet schools operate six days a week), or that statistical procedures are used to build a rational argument in research (Soviet researchers rarely rely on statistical reasoning). Other differences between cultures are more subtle and illuminate problem areas in "our" work rarely addressed. I would like to illustrate this with two examples.

*Example 1: The conflict between individualism and collectivism.*

Schooling is of necessity a collective experience. For the child, being in school means being in a crowd. For the teacher it means always being responsible for a group of students. Thus, the problem of how a small number of adults can organize and manage a large number of children is the central organizational problem of schools. However, within this collective experience two aspects of the doctrine of

individualism in American schooling are paramount. First, we believe that students differ on a variety of important intellectual traits, and that these differences should be considered in schooling. The implication is that, by attending to such differences, instruction will be both more efficient for the institution and more rewarding for each student. Second, regardless of the ingredients of a lesson, the dominant pedagogy in our schools involves intragroup competition. Competition is fostered because it is both a method of motivating learning and a means of differentiating between students. Accomplishing an assigned task and perhaps doing it better or faster is rewarded. Also, going to school means being evaluated. Children are constantly having their words and deeds judged by others. Thus, the job of teaching is in large part seen as creating competitions and judging the results.

Before going to the Soviet Union, I had always taken individual differences and intraclass competition for granted. It is true that we have not provided for individual differences very adequately and the competitions often are biased, but we are striving via our research and reform movements to overcome our inadequacies and our biases.

In the first classroom I visited in Moscow (a third-grade class doing multiplication problems), I noticed a child copying the steps of a problem solution from his desk partner. I commented to my guide that the child was cheating. The reply was, "Cheating? No, he is being <u>helped</u>." This turned out not to be a unique experience. In fact, deskmates were working together in all classrooms I visited, copying from each other, assisting each other with answers, and so on. Intraclass competition seemed not to exist. Instead, collaboration seemed to be the rule.

I raised questions with Soviet researchers about these observations. They in turn led me back to points raised in the Soviet papers in the seminar. For example, Lerner stated:

> ... that the teaching process is a social-pedagogical category essentially given by the social system, it follows that the most vital link between the curriculum and education--upbringing process is that which represents the needs and goals of a society vis-a-vis its schools. Educational systems differ according to how they answer the following four questions: for what, whom, what and how to teach. In other words, the answer to the questions of educational goals, student contingents, curriculum and teaching methods taken collectively determine the essential features of any education. The USSR is called upon to produce roundly and harmoniously developed persons capable of participating in the building of the just social system; it furnishes equal educational opportunities for all social strata. ...

He goes on to state that "the all-around development of personality is the global goal of schooling." An appropriate personality development involves such principles as:

-- formation of materialist outlook at the scientific-theoretical level
-- humanistic orientation of the content
-- formation of progressive social ideals and a wide system of humanistic values, including moral and aesthetic ones
-- fostering of optimism and conviction of progressive development of mankind
-- inculcation of qualities conducive to a socially active personality
-- inculcation of patriotism and internationalism
-- observance of the polytechnical principle in the content of education
-- observance in the content of education of conditions providing for readiness for the world of diverse work
-- formation of a creative personality
-- formation of a physically fit personality.

Krajevskij stated that these principles could only be accomplished by

> ...rearing the class as a learning collective which furnishes conditions conducive to both active and well-organized work of all pupils.... ...school children study in class collectives. Following graduation school-learners work also in various collectives which are linked through numerous channels to the great collective of our entire nation.

During the seminar, I had viewed these statements as being in the nature of public-relations rhetoric. After visiting schools, I see them now as operating principles.

Soviets recognize individual differences, but do not view them as that important. Schools should not try to differentiate between students. Being a group member of the collective; helping each other; not standing out or being different; minding parents, teachers, party heads, etc.; not complaining; and so on, are valued. Competition (for grades, class standing, etc.) is viewed as a "capitalist" strategy to train students to be better salesmen and consumers. Collaboration is viewed as a socialist strategy to train students to be better members of collectives. Some of the consequences of this fundamental difference between Soviet and American schools have been commented upon in Chapter II. Soviet schools are warm, caring places; college preparation is not central to schooling; parents are involved.

This experience has made problematic the individual differences—intra-class competition basis of instruction I have taken for

granted. Contrasting schools has made real to me the notions of the "hidden curriculum" of schools and the importance of cultural ideology on schooling.[4]

I might add that lack of classroom differentiation has created a serious problem for the Soviets: namely, how to identify talented students. They provide special training for some talented students, but the opportunity for such training depends heavily on parental occupations and on where one lives. A child of professional parents in Moscow or Leningrad has some chance of attending specialized schools. Children in rural villages whose parents work on collective farms probably have little opportunity for such experiences.

*Example 2: The work of teachers.*

In American elementary schools, children are initially grouped by age (all six-year-olds are in the first grade). In most schools, another step in grouping is taken by subdividing the children into sets containing 20 to 30 members and assigned to a teacher for a school year--the self-contained, age-graded classroom. The teacher, within some general constraints involving grade-level expectations, is given considerable latitude in scheduling and deciding what will be done. The work of a teacher then becomes: selecting and assigning lessons to a class of students, starting and stopping the lessons according to some schedule, explaining the rules and procedures of the lesson, judging the actions of the students during the lesson, and throughout maintaining order and control. This the teacher does for all areas of content for the same group of students for a school year. A key assumption made in the American system is that each teacher is an independent decision-maker. Although lots of materials, curriculum guides, schedules, pupil data, etc., are available, the independent teacher decides what content to teach, what activities and materials to use, how students should be organized, how lessons should be presented, how performance is to be judged, and so on. Creation and adaptation of lessons for the particular group of students under the teacher's direction is the basis of the job. An obvious consequence of this practice is that the details of what goes on in each class are different.

In visiting eleven classrooms in three Soviet schools, I found striking similarities in what was going on across classes. Beginning in grade 4, classes are taught by subject matter "specialists": mathematics is always taught by somebody who teaches nothing but mathematics. From grade 4 upward, the self-contained classroom does not exist. Furthermore, each class is taught in the same way in every classroom throughout the country. Teachers follow a prescribed didactic sequence involving first a lecture and then questions. The prescription of what to do is in the form of a detailed syllabus which is almost a script the teacher is to follow. Students proceed

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[4] Two good references on this topic are Apple (1979) and Cagan (1978).

in somewhat rote fashion upon the prodding of teachers. In this instructional system the teacher is viewed as a conduit. Teachers are not supposed to adapt or change materials to meet the needs of their students unless they get prior approval. They are not seen as professional decision-makers, responsible for planning a curriculum.

This realization has made problematic the latitude we in the United States give teachers in deciding how instruction is to proceed. This is of particular concern to me since, for the past several years, I have been involved with several projects in which what teachers do (teacher actions) is being related to what students do and to their achievement.[5] From these classroom studies, it is agonizingly plain that the teaching of mathematics by many elementary school teachers in the U.S. is awful. Little time is allocated to mathematics; structuring of lessons, higher-order questioning, and appropriate feedback are infrequent; students spend inordinate amounts of time waiting or off-task. The Soviet experience has led me to question our tradition of allowing teachers wide latitude in instructional decision making. When latitude leads to license to be incompetent, can we afford it?

In summary, for social scientists it is important to examine schooling in different ideological settings. Only by doing so can we discover that our assumptions are not invariant across national boundaries.

### *Answer 2*

My second answer to the question of "What can we learn...?" is that comparative studies can *illuminate procedures used by different systems to solve the same problems*. In so doing, the comparisons can reveal the strengths and weaknesses of each system. To illustrate this let me again use two examples:

*Example 3: The Development of Curricular Materials.*

In all countries the preparation of content materials to be used by teachers and pupils in classrooms is a major enterprise. In this country there are two systems (sometimes complementary and sometimes antagonistic) involved. The first is the foundation-sponsored (usually with federal funds) development groups such as the University of Illinois Committee on School Mathematics (UICSM), School Mathematics Study Group (SMSG), Madison Project, Minnemast, Comprehensive School Mathematics Project (CSMP), Individually Prescribed Instruction (IPI), Developing Mathematical Processes (DMP), etc. In these groups

[5]The studies are the Beginning Teacher Evaluation Study conducted by the Far West Laboratory, and the IGE Evaluation Project and the Integrated Studies in Mathematics Project, both conducted by the University of Wisconsin R & D Center.

mathematicians work with teachers and often other educators to plan, write, and tryout sets of materials. An ideal sequence of research, development, formative evaluation, production, implementation, and summative evaluation is expressed, even if it is rarely followed. The second system is the commercial publishers. Since schools buy materials, competing for those orders is big business. Publishers prepare materials in a variety of ways, sometimes by producing foundation-sponsored materials, more often by adapting such materials, and most often by having "in-house" authors prepare the texts. The end result is a wide variety of materials being used in schools with considerable variability in what content is included, how the content is approached, and what teachers and pupils actually do with the materials.

In the U.S.S.R. the system for producing materials is quite different. Educational planning follows a clear, deductive sequence. Official planning begins with one societal objective for schools--to produce good citizens. Given this goal, the organization of educational planning and research follows deductively. The writings of Marx and Lenin are used to outline what it means to be a socialist and the attributes of how to become one. Then a series of principles related to learning and instruction are posited, based on such authors as Vygotsky, Galperin, and Leontiev. These are psychological principles of communist upbringing. Next, the structure of the content to be taught is specified.

It is only here that mathematicians and mathematics enter. The need for mathematics must be first justified as socially useful. As Zverev states it:

> The December, 1977, Resolution of the Central Committee of the CPSU and the USSR Council of Ministers on School emphasizes that the improvement of the content along with the other components of education must meet practical needs. It also stresses the task of combining thorough knowledge of the fundamentals of science and work training with preparation of school students for socially useful labor.

To achieve such school improvement it is the responsibility of the Ministry of Education and its Academy of Pedagogical Sciences

> ... to take into account the conditions under which modern school operates, for example, the acceleration of scientific and technical revolution, high life pace, intellectual character of labor, wider professional outlook, the necessity for new, more complicated activities. These demand better educational and polytechnical training, quick orientation in the mounting scientific and political information, combining of knowledge of the scientific fundamentals of production and formation of labor habits, polytechnical training.

From this basis Lerner states that the first principle which determines the design of curriculum in the socialist school is "the scientific nature of content." By this he means:

> ... the fundamentals of sciences central to the contemporary natural-scientific and social picture of the world. The fundamentals of sciences are seen as the sum total of fundamental concepts, laws and theories and basic facts they condition, as well as major types of problems tackled by the science, and its methods...

The responsibility of the mathematician then is to specify the concepts, skills, problems, and methods of mathematics, and to help the psychologists and pedagogical scientists in their translation into lessons. Note that there is less scope for intellectual disagreement as to what constitutes an appropriate conceptual framework than we are accustomed to in the United States, where, for example, the BSCS produced three different biology courses, and where three alternative geometry programs are based, respectively, on Legendre's, Hilbert's, and Birkhoff's axioms. Such arguments are interesting to Soviet scholars, but if a single program is to be developed, then agreement on fundamentals must be reached before pedagogical principles can be derived. Within the Soviet framework, this agreement must be nation-wide. Finally, based on this logical superstructure, the Academy of Pedagogical Sciences develops instructional programs.

At present the basic psychological structure for the teaching of mathematics has its roots in Vygotsky's seminal work on proximal zones of learning and several interpretations of that theory for mathematics. Davidov's paper for the seminar outlines first the work in the 1940's by Menchinskaya (also a member of the seminar):

> ... one of the key moments in the child's learning is the formation of the child's generalized modes of operations over the obtained knowledge. These generalized modes allow children to apply these modes in varying conditions within frameworks of a definite group of objects of common character (papers by D. N. Bogoiavlensky, E. N. Kabanova-Meller, S. F. Zuickov and others). Mastering of the generalized modes of operations is an important means in the child's mental development (alongside this process the principle of the unity of education and mental development is realized.)

Davidov goes on to state:

> ...In the 50-ies another approach has been outlined and led by P. I. Galperin. The main achievement received in the frames of this approach is the assumption of basic types of learning and stage-by-stage formation of mental actions as a theoretical consideration of psychological mechanisms of concept and skill formation. Among all the variety of types

and forms of learning it is possible to single out the three main ones.
The first type of learning implies a spontaneous orientation of the child in the situation of learning, when mental actions are acquired without a specially organized presentation of the entire system of those signs of orientation which similarly guide the child in realization of actions.
The second type involves the discovery of such a system by the school child but with the help of a teacher. And finally the third type of learning suggests that the child himself discovers the general method of forming the entire orientating basis for the required actions with the indirect prompts by the teacher.
...Over the last years on the basis of this approach an attempt was made to imply some principles of the general theory of control. The aim of this attempt was to reveal conditions of effective control over the child's cognitive activity and control over the process of learning proper (papers of Talizina and others).

Then in the 1960s:

...another trend was formed led by D. B. Elkonin. In the frames of this trend there are studied conditions and regularities of the child's specific learning activity, which has its own needs, motives, tasks, actions and operations. The motive of the learning activity is the necessity of creative approach to the reality. When the child solves a problem, he masters the universally theoretical method of solving the entire class of problems. This general theoretical method is based upon the analysis of inner conditions of the origin of the given system of objects. The child reveals the genetically initial universal relation which lies in the basis of all the particular manifestations inside the system. This generalization differs from the formal empirical generalization based on the comparison of external similar indications of a group of objects. Such a comparison allows children to acquire only the ready knowledge of a descriptive character, while analysis and theoretical generalization reveal the sources of the origin of the notion of an object. This theoretical generalization introduces the child into the corresponding theory. Solving of a learning task is carried out with the help of following actions:
1. transformation of the situation which may bring about a discovery of the universal relation proper for the given system of objects under analysis.
2. modelling this relation in a graphic or a symbolic form.
3. transformation of the model to studying properties of the universal relation in the general aspect.
4. deducing the series of particular concrete tasks which may be solved by the general method.

5. controlling fulfillment of the preceding actions.
6. estimation of the general method as the result of solving the given task in the process of learning.

Although the principles are stated abstractly, all of these psychologists have primarily worked in the area of mathematical learning. This list of six major actions is their attempt to build a set of psychological principles which can be used to develop programs.

In the end, this planning sequence results in a syllabus for mathematics for use by teachers and students throughout the country. *The syllabus is a very detailed lesson plan (not quite a script) which teachers are to follow. For example, a syllabus includes the kinds of questions teachers are to ask in each lesson and how much to cover each day.* Soviet planning, their development of syllabi, and their research show this same pattern for central control and tight organization. The Ministry of Education, in response to political priorities (established by the Politburo, etc.), establishes goals (five-year plans). It assigns to the Academy of Pedagogical Sciences all tasks associated with curriculum development, teacher training, and research. These tasks are then parcelled out to the 13 different institutes within the Academy. These institutes then work on their assigned tasks (often with the cooperation of experimental schools).

In summary, curriculum development is a top-down bureaucratic system. It is the responsibility of the Academy of Pedagogical Sciences to see that the syllabi that will be used in the schools are developed. For example, the development of the last new mathematics curriculum required more than 10 years to produce.

Clearly, the systems of curriculum production in the U.S. and the U.S.S.R. differ. By contrasting them, it is obvious that the strengths of one are the weaknesses of the other. For example, in the American system it is easy to introduce new ideas into curriculum production. There are no constraints on trying out, producing, or selling anything. Thus, as times and technologies change, materials can be quickly produced. In the Soviet system this is impossible. New ideas must be approved, carefully examined, tried out, and developed by the Academy before they can be used in schools. However, the American system has no real quality control. New material may be nonsense or use terrible pedagogy, but if schools are willing to buy, then it is sold. Likewise, new good programs have problems getting materials adopted, if the costs are too high. Thus, many new (particularly federally funded) projects find publishers unwilling to publish and market them. This is certainly not the case in the Soviet Union. Quality control is of critical importance, and if materials are approved then <u>all</u> schools across the nation will use those materials.

A second weakness of the American system is the lip-service we give to research principles. That is, materials may be produced with little or no tryout in classrooms by teachers and may be

totally inappropriate. Again, this is certainly not the case in the Soviet Union. The research paradigm is followed carefully, believed in, and if the material is appropriate it is then implemented with the full resources of the Academy of Pedagogical Sciences.

*Example 4: The Logic of Research*

American educational research is hard to portray adequately, in part because its philosophic basis is as varied as the philosophies of the individual researchers. The inadequate distinction between "pure" and "applied" research separates inquiry into two categories: theory-based inquiry aimed at clarifying propositions within some hypothesized framework, and engineering studies aimed at building better materials or techniques. Other distinctions, such as "conclusion-oriented" vs. "decision-oriented," or "quantitative" vs. "qualitative," merely highlight the variety of activities labeled as research in this country. In spite of these distinctions, the logic of American educational research most often involves the language of experimentations and the idea of cause and effect from a "critical-realist" philosophic position.[6] The desired end-product of a large proportion of American research is meaningful causal assertions which are contingent on many conditions and hence fallible and probabilistic. Thus, many U.S. researchers commonly rely on such ideas as validity, reliability, and generalizability. The terminology and methods of statistics are used to build logical arguments about causation and generalizability.[7] Most researchers today follow hypothesis-testing procedures best characterized in terms of Popper's notions of falsification (1972). Researchers posit hypotheses about relationships between variables, and then systematically attempt to falsify those propositions. While there are countertrends within the American research scene, this "realist" orientation seems to be the dominant language and methodology employed by educational researchers in this country.

In the Soviet Union, research follows a different logic. As perhaps nowhere else, in the U.S.S.R. philosophical considerations have strongly influenced psychological theory; psychological theory has, in turn, helped to determine educational practice. Educational method, in other words, has been explicitly justified in terms of the major characteristics of the Soviet view of psychology which in turn has been justified by a coherent and unified set of underlying philosophic principles. This interdependence is perhaps the most striking characteristic of Soviet research in education. Their

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6 [Ed. note: Alternative foundations of research have been stated by Herbert Simon, John Seely Brown, and others. These are probably minority views, but they are stated by researchers of impeccable credentials.]

7 An excellent book on this is Cook and Campbell, Quasi-experimentation Design and Analysis (1979).

research thus follows a basic Aristotelian notion of logical verification. To illustrate this dependence, let me briefly summarize the major characteristics of current thinking in the Soviet psychology and its philosophical determinants.

In general, psychology in the U.S.S.R. is marked by the following major characteristics: (1) a strong emphasis on the importance of practical activity in an objective physical and social world for developing knowledge; (2) a concomitant emphasis on the importance of conscious knowledge in directing human activity; (3) a dialectical, genetic perspective which argues that intelligent human action can only be understood in its development, and that development proceeds by stages and as an active process; (4) a socio-cultural perspective which emphasizes the importance of adult-child social interactions (particularly those involving the linguistic systematization of cultural knowledge) in the determination of the child's developing intelligent activity; (5) a neuropsychological perspective which dictates that whatever neurological knowledge exists must be taken into account in understanding psychological phenomena, but without in any way reducing the psychological determinants to underlying neurological ones; and (6) a complete rejection of standardized intelligence testing in favor of a clinical approach to individual diagnosis.

(1) Practical Activity in Developing Knowledge. Soviet philosophy assumes both the existence of a real material world independent of any knowing subject and the progressive adequacy of both the individual's and the society's knowledge of that world over time. Both the source and the criterion of adequate knowledge is practical activity. Human action in a real world develops and corrects man's knowledge. For psychology, this implies that an understanding of the characteristics of human thinking can be obtained through an analysis of intelligent activity. The structure of human action becomes the structure of human thought. For education this emphasis on action means that diagnosis of the nature of a child's development must rely on an analysis of his or her patterns of activity and that instruction must include the actual manipulation of concrete materials in meaningful, ecologically valid situations. Thus, academic programs must be designed around practical, life-like experiences for the child if, in the Soviet view, they are to attain their objectives of developing the child's cognitive capabilities.

(2) Conscious Knowledge in Directing Human Activity. Just as practical activity serves to correct and develop human consciousness, conscious knowledge is seen as guiding and directing the activity by which a person alters his or her own environment and consequently becomes capable of self-development. For Soviet psychology, this implies that the study of behavior by itself, apart from a description of the knowledge which guides and directs that behavior, is fruitless. A proper understanding of human action can only come through study of that action as intelligent knowledge-guided action. This implies that instruction which attempts to deal only with

surface behavior while ignoring the child's understanding of what is expected to him or her in a given situation is to be eschewed. For behavior to be deemed adequate and intelligent, the child must comprehend what he or she does and why he or she does it.

(3) Dialectical, Genetic Perspective. Dialectical materialism is founded on the assumption that all that exists exists in a state of constant alteration and development, and that the form of this development is described by three laws (called the laws of the dialectic). Very briefly, according to these three laws development is both continuous and discrete. It consists of gradual quantitative changes which give rise to sudden qualitative changes in which phenomena become determined by new sets of laws to which they were not previously subject. Such development occurs as a process of the resolution of internal contradictory tendencies in a phenomenon. Hence the motive force for development lies within a phenomenon rather than in the surround. Lastly, development not only occurs by stages, but new stages integrate previous stages so that characteristics which exist at a lower stage reappear at higher stages in a continuous progression.

The implications for psychology of the notions of constant alteration and of development corresponding to these general characteristics are several. First, in such a system all psychology must, in an important sense, be developmental psychology. Psychological phenomena can only be properly comprehended in the process of development. Secondly, psychological theory must to a large extent be stage theory. Since development is qualitative as well as quantitative, psychological phenomena are best understood in terms of relatively constant, integral unities at particular points of a genetic process, or stages. Lastly, psychology must reject models of human intelligence which see it as a passive receiver of sensations, striving to regain a state of equilibrium. The psychological organism is an active unity which carries within it the motive force of its own development.

These views may be discerned in a number of areas of Soviet work with children. For example, the severity and the source of handicaps are classified on the basis of qualitative differences in performance in complex tasks rather than on quantitative differences such as the number correct on a psychometric instrument. In the design of curriculum materials, the principle of constant alteration and development leads Soviet educators to adopt the strong attitude that development never ceases, even for a child whose progress may be extremely slow. In addition, development is seen as stagebound, progressing through a number of "zones." The task of the teacher is to help the child systematically lead himself or herself through this developmental progression. The notion of the child "leading himself" with adult help also comes from the principles of dialectics, which assert that development is an internal principle. Development is in fact self-development, and one of the major emphases in Soviet educational work is to assist the child in achieving the means to continue developing himself or herself.

(4) Socio-Cultural Perspective. The notion of group labor, out of which society, language, and culture evolved as means through which humans can change the world and consequently alter and develop their own consciousness, is of primary importance in Marxist thought. From this perspective, Vygotsky developed a socio-cultural theory of cognitive development which continues to influence most of special educational practice in the Soviet Union. Specifically, Vygotsky noted the importance of what he termed "cultural mediators" (of which language is the most important), which are employed in social intercourse as socially developed aids in relating to reality as the primary means by which the child comes to regulate his or her own higher mental functions. Words and other social mediators possess the unique characteristic, in Vygotsky's view, that they are always a 'sign,' a reflection of something. It is in employing such signs that humans become capable of easily introducing changes into external reality which in turn reflect back upon and develop their consciousness. By altering his medium, a man is able to regulate his own behavior and control his own psychic functioning. He is no longer dependent on the reality of the external situation.

In discussing the ontogenesis of this regulatory process, Vygotsky asserted (and Soviet theory continues to rely heavily on this notion) that all specifically mediated human mental processes arise only in the course of social activity, in the process of cooperation and social intercourse. Psychological functions at first shared between two people, in particular between a child and an adult, become the internalized psychological processes of one person (in particular the child). Thus the structure of mental processes is at first present in humans' external social activity and only later becomes internalized as the structure of inner mental functions (e.g., egocentric speech is internalized as inner speech or verbal thought).

These notions figure prominently in both the classification and training of Soviet children. For example, one major method of distinguishing among the three categories of functioning--"normal," "developmentally backward," and "defective"--is to present the child with a task which requires that he or she supply missing organization to materials. In such a situation, appropriately aged children who are functioning normally will be capable of providing some of the missing organization themselves, but retarded children will not. If, however, the same task is then presented again with increasing levels of adult organizational intervention (in which the adult provides the child with certain prompts in an attempt to help him or her to organize the information in the task), the "developmentally backward" child, who may perform much more poorly than the normal child without such prompts, is capable of improving his or her performance virtually to a normal level through utilization of this additional organizing information. The "defective," on the other hand, will generally be unable to take advantage of the increased social organizational information to increase performance.

(5) Psycho-Neurological Perspective. Psychological phenomena are seen from the Soviet perspective as very closely linked not only to the real physical world which the knower gradually comes to know, but also to underlying neurological function (referred to as "higher nervous activity"). In psychology, this is manifested in the high level of development reached by Soviet psychoneurology, in which various types of behavioral dysfunctions may be quite reliably linked to respective forms of organic brain damage, and in a reliance on many of the ideas of Pavlov in the formulation of psychological theory. Soviet psychologists are careful to stress that psychology cannot be reduced to physiology; but physiological laws and facts must be taken into account in any adequate psychological conceptualization.

In Soviet education, this perspective is perhaps most clearly reflected in the lengthy and careful clinical diagnostic procedure through which the organic etiology of particular handicaps in children is diagnosed. As a rule, such a diagnosis always includes medical and psychoneurological examination in which the possibility and localization of organic etiology is explored. This information is then employed along with more specifically psychological information about the child's action patterns to determine placement and to a large extent even the course of training. In addition, under the influence of the psychoneurologic and Pavlovian perspective, Soviet training programs stress the notion of "compensatory mechanisms," a notion which derives from the idea that portions of the brain which are functioning adequately may in time take over some of the functions normally assigned to brain areas which have received partial lesions.

(6) Lack of Standardized Testing. For reasons derived from the philosophical positions discussed above, Soviet theorists reject the use of standardized intelligence, aptitude, or achievement assessment. Standardized tests, they argue, rarely stress appropriately the child's active manipulation of meaningful objects. Rather, such tests, for purposes of standardization, must place the child in what Soviet psychologists consider highly rigid, static, and artificial situations which deny children the flexibility of utilizing the skills that they do possess to compensate in part for those which they do not possess (and hence, the Soviets would claim that the examiner finds out very little of interest about either). Furthermore, standardized tests result primarily in quantitative rather than in qualitative assessment of the individual child and hence stress the continuities in development at the expense of the equally important discontinuities. Lastly, standard psychometric testing drastically restricts the form of or even entirely eliminates adult-child interactions; in general, the procedure thus leads to a tendency to fail to take account of important psychoneurological information.

In place of standardized psychometric evaluation, Soviet psychologists and educators employ a multi-faceted, clinical assessment in

which doctor, psychoneurologist, speech therapist, hearing specialist, teacher, or whatever other personnel might be indicated examine the capabilities of the child and arrive at a consensual diagnosis.

The logic of Soviet research is to conduct studies with this framework of philosophic-psychological thought. Each study is seen in terms of adding a piece in a larger chain of inquiry about a particular phenomena from this perspective. The Soviet researcher is not challenging assumptions, but is trying to verify assertions developed from the theory. It should be noted that the typical articles written by psychologists are either descriptions of instances of how materials are used from selected tryouts of topics to verify the principles from which curriculum materials were developed, or present information on how ideas are processed by children within a particular setting. In this respect there is no concern about problems of sampling, statistical inference, ruling out biases, or alternate explanations to particular hypotheses. Since children are not basically different, any sample of children will do as well as any others. And if results are not verified in a particular teaching experiment, then it is assumed there is a flaw in the derivation of specific activities.

In comparing these two systems of research, the strengths of the American system seem apparent. First, the possibility of continually questioning the assumptions upon which a research question is based is central to American research. Thus, we are engaged in a proliferation of models and methods for conducting research and a continual healthy argument about the assumptions upon which a research sequence is based. This is far less prominent within Soviet methodology. For the Soviet researcher, assumptions are philosophically based, within a particular framework. One does not ordinarily question that framework. Second, researchers in the United States are concerned about validity, generalizability, sampling, etc. These have become cornerstones of American methodology, although we often overuse statistics, or seem to equate the use of sophisticated statistical techniques with good research. The Soviets see little rationale for using any statistical argument to justify the probability of a particular result. On the other hand, our weaknesses are also apparent. The lack of systematic chains of inquiry which tie together the results of several studies is a particular concern. This simply could not happen in the Soviet system. Research would not be authorized unless it did tie in with prior ideas. Our reliance upon agricultural models sometimes clouds our view of the underlying mechanisms of learning and teaching. Too often our research is at a surface level rather than at an underlying level. It thus relies primarily on statistical procedures for its validity.

In summary, these two examples should illustrate in part the distinctions between processes used within the educational research systems of the two countries. By examining such processes, one can illuminate the strengths and weaknesses of each and in turn begin to understand and appreciate both systems in more detail.

*Answer 3*

My third answer to the question, "What can we learn..:?" is, *comparative studies can reinforce one's understanding of common contemporary problems in education.* Let me comment on what Zverev has indicated are four major problems which need to be addressed by the Academy of Pedagogical Sciences.

*Problem 1: The development of creative thinking.*

> Future progress in our society will, to a great extent, depend upon what we teach in school and how we teach it. Our task is to teach youngsters creative thinking to prepare them for life and practical work, said Breznev in 1970. The Soviet school is faced with the principle task of orienting youngsters towards socially youthful activity motivated by communist consciousness and devotion to the lofty moral ideas of our society and at the same time developing their intellectual faculties in creative potential.

In this statement the conflict between wanting citizens to conform and at the same time to be creative is apparent. This is the same strongly held, conservative position voiced during the past quarter century in the United States. Socially, within the United States, the 1950s were a period of political turmoil and technological upheaval. The "cold war" and the Korean war, when added to a potential nuclear holocaust, kept both military preparedness and the continual development of sophisticated armaments at a level of high national priority. Conservative intellectual retrenchment led by Senator Joe McCarthy and the emergent civil rights movement were two focal points of the political turmoil of the era. This helped form the "conservative" side of the tension. On the opposing side, pressures for creativity continued to build. The rapid development of computers, along with a series of spectacular basic inventions such as the transistor, were creating new probabilities for giant corporations and reshaping most other industries. The need for a large cadre of scientifically trained personnel was critical, and was argued for primarily in terms of national survival.

Schools fit into the arguments of this period of American educational discourse in three ways. First, one set of educational critics claimed that few of the graduates of American schools and colleges had an adequate mathematical, scientific, or engineering background. The culprits were seen to be the progressive education movement, the life-adjustment curriculum, and in particular education professors. The antedote was seen as the development of curricula which emphasized intellectual training through the academic disciplines. During this time several study groups (such as the University of Illinois Committee on School Mathematics, organized in 1952) began to produce some curriculum materials which emphasized the structure of the disciplines. When the Soviets launched the

first space satellite in the autumn of 1957, a shocked United States became aware that its technological supremacy was being challenged. What followed was the "modern curriculum" movement which involved the federal government spending considerable sums to have new discipline-oriented materials developed and to have teachers retrained. The new curricula were deliberately developed under the direction of scholars from the disciplines. Experienced classroom teachers were junior partners in the endeavor. Educationists were expressly excluded.

A second group of educational critics argued that schools were psychologically alienating. Spokesmen for low-income and minority groups, for example, pointed to a pattern of unsatisfactory achievement by school children from low socio-economic and minority backgrounds. It was such children who were most likely to score poorly on tests of achievement in reading and mathematics. There were insistent demands made that schools become more accountable for the learning of all their pupils, including low socio-economic level and minority children.

The third aspect of the educational debates was less a criticism of schools and more a prescription of how to produce a better system. American belief in science and technology had reached a peak in the years following World War II. Many believed that the same rational procedures that enabled the military and industry to conduct a massive war and then provide for an affluent consumer society could be enlisted to solve the most pressing social and political problems that confronted the United States.

Throughout these debates, while there was the explicit desire for a better and more equitably trained citizenry who could think and solve complex problems in creative ways, there was also a simultaneous implied desire to "preserve democratic ideals," "keep God in the classroom," "reinforce the values of home and family," etc. The implication, then, is that it is important to challenge the assumptions that underly scientific principles or engineering procedures, yet one is not expected to challenge the assumptions upon which the social system rests. That is, citizens need to be technologically creative but socially accepting. Clearly, in the U.S. this has not been the outcome of the past two decades. Our citizens today are not socially accepting.

In summary, in both the U.S. and the U.S.S.R. there is a growing realization that ideological rigidity can impose thought restrictions and in turn hamper creativity; yet fostering creativity which allows citizens openly to challenge assumptions may also undermine the ideological basis of the society.

*Problem 2: The determination of a didactic system of concepts in the basic relations within content areas is of paramount importance.*

The Soviets recognize that it is important to follow the structure and main trends of a scientific discipline in order to obtain a valid education. However, no branch of science allows for a single approach nor has a single structure, which complicates the problem of establishing the structure of school subjects. Thus, content coordination and integration becomes a critical problem for the Soviets. Sound familiar? It should; the same concerns have been repeatedly voiced since the start of the curriculum reform movement in this country in the 1950s. We too have not solved the problems of content coordination and integration.

*Problem 3: The further improvement of the new educational content, particularly in mathematics.*

The new content in Soviet school mathematics is characterized by a higher generalization level, a firmer logical and theoretical basis, an early introduction to theory, a combination of deductive and inductive methods of material presentation, greater polytechnical knowledge of modern production, reduced reference material, and more assignments providing for students' general development. However, within this system, the problem of excessive material both in the syllabi and textbook has not been solved. In fact, Soviet pedagogical scientists are faced with the task of eliminating "excessively difficult material of secondary importance" (December 19, 1977, resolution of the Communist Party Central Committee in the U.S.S.R., Council of Ministers, on further improvement of instruction and education of general school students and their preparation for labor). In particular, the Soviets are concerned with:

1. the avoidance of description in material presentation, cutting down on facts and reference material;

2. reduced terminology in some of the lessons of the textbook (a particular problem in mathematics texts in early grades);

3. cutting down on complicated scientific material while preserving a theoretical level and the educational role of the subject;

4. elimination of material rendered impractical by the highly prohibitive nature of teaching devices required for its presentation;

5. better methods of teaching certain concepts, theoretical propositions, etc.;

6. careful selection of objects of study for the sake of greater generalization and stronger links between theory and practice; and

7. doing away with duplication in view of interdisciplinary overlap.

The U.S.-Soviet discussions on these points with respect to mathematics indicated some of the same problems we in the U.S. are facing in terms of the "back-to-the-basics" movement. It was openly admitted that the current mathematics materials were too formal, too abstract, and too difficult for many students and too difficult to teach for many teachers.

*Problem 4: The development of moral convictions.*

The Soviets express a strong desire to have students taught to defend moral convictions, to prove one's views, to make sacrifices for the sake of ideological truth, to comprehend the ideological civic meaning of knowledge, and to have a sense of uncompromising opposition to that which is immoral. There is a strong feeling that today's youth are being corrupted by immoral influences from the West and that the sense of ideological commitment derived both from the Revolution and from World War II are being lost. Young children are too materialistic and not idealistic enough with respect to the foundations of Soviet citizenry. These questions are very similar to the kind of questions being asked of American education concerning the preservation of American ideals, back-to-the-basics, standardizing general education, a commitment to democratic ideals, and so on.

In summary, the teaching of mathematics in the Soviet Union differs from the teaching of mathematics in the United States. Our research processes, planning procedures, and curriculum development have their counterparts, but with nuances and philosophic differences that must be understood. Only by examining different ideological systems can we develop laws of the social structure, human psychology, and education which cut across national boundaries. Findings in one society must be tested against the experience of other societies. Variation through time and across cultures can be turned to scientific advantage. Thus, for the social sciences in general and mathematics education in particular, we must proceed by careful examination of the education in diversified cultures like that of the Soviet Union so that our understanding of the educational process can be enriched.

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# Chapter VII

## Soviet Approaches to the Study of Problem-Solving Processes in Mathematics

Sidney Rachlin

### A. A Fable

Many stories have been told about a foreigner's first visit to a baseball game. In mine, the visitor's name is Nikolai. Nikolai had been sent to the United States to study the American phenomenon of baseball. After two days of intensive observation, Nikolai returned to his native land to report on his investigation. Here are some excerpts from his written report.

> Each session began with a token tribute to the government. The tribute, though apparently unrelated to the activity, was nonetheless evidently essential to the process of beginning the activity.
>
> As has been reported in the past, the Americans are a very class-oriented society. This was again reinforced by the distinction in the subject's attire. The representatives of the ruling class were dressed in black suits, stood throughout the session in control locations, and shouted orders to the working class. The working class were dressed in either grey or white numbered uniforms. Absenteeism must run very high since although I observed one subject in a grey uniform numbered 72, there were only about 30 workers in grey uniforms present on either day.
>
> It was difficult to focus on the entire phenomenon at any one time. Other than styles of clothing, there were very few commonalities observed; e.g., there was no apparent pattern for the number of people on the playing area, nor for the length of time each group was on the playing area.
>
> Quantitative evaluation of the phenomenon revealed that the grey group scores decreased from 13 to 1 while the white group scores remained a constant 2. The mean score for the grey group was 7 and the mean score for the white group was 2. Thus the grey group appears to be a significantly better group. Yet someone said the groups were now even in the series.
>
> Even my attempts to meaningfully focus on an individual provided some confusion. While observing number 72 on

the grey team, I noted that he hit the ball with the stick nearly 1200 meters. Despite the fact that he hit it for that distance and was still able to aim it right at one of the other players, the spectators booed. Yet later he hit a ball not 30 meters and the crowd cheered him.

My recommendation is that further research needs to be done to fully understand the phenomenon. Perhaps future studies should be of longer duration. One-on-one interviews with some of the subjects might prove beneficial. Or, better yet, why not have some of our people exchange with American counterparts; then our people could participate in the phenomenon.

This chapter focuses on Soviet approaches to the study of problem-solving processes in mathematics. Some implicit parallels with Nikolai's exploration will become apparent. Foremost is the need for clear and concise definitions, illustrated by Nikolai's difficulties in interpreting certain terms. We shall therefore begin by discussing some working definitions to serve as a common thread woven through the tapestry of Soviet problem-solving research.

*Definitions*

A *problem* is defined as a "task" which a "subject" attempts to resolve, given that this resolution is within the subject's ability and is not resolved by the subject's immediate application of some algorithm. Resolution of the task is taken as the subject's belief, stated or implied, that he or she has obtained the "actual" solution. Hence, whether or not a task is a problem is dependent on the characteristics of the subject and his or her attempted paths to resolution (Menchinskaya, 1974; Kantowski, 1975; Kilpatrick, 1969).

A "task" which a "subject" resolves through the immediate application of some algorithm is called an *exercise*. An *algorithm* is an unambiguous series of steps which completely specifies the resolution of a task. By contrast, *heuristics* are neither unambiguous nor complete. They must be supplemented by more specific procedures at the level of implementation, and they come with no guarantee whatsoever (Landa, 1975).

In each mathematics class there exist some students for whom only the first few tasks of a worksheet are really problems. After solving these few examples, the students generalize an algorithm which they apply to succeeding problems. In many classes there are also students for whom each task of a worksheet remains a problem. Hence, for some students "verbal problems" become exercises under generalized headings such as "motion towards--motion apart," while for other students a task such as "factor $9 - 4x^4$" bears no resemblance to previously solved problems of the form $a^2 - b^2$. A task which is "one student's problem, may be another student's exercise, and a third student's frustration" (Henderson and Pingry, 1953, p. 232).

The distinction between *algorithm* and *heuristic* is equally intimate. Heuristics, according to Pospelov, Pushkin, and Sadovskii (1972), is "the science which studies the laws governing the design of new actions in new situations." But "new" is relative to the individual. Situations and actions are new if they are perceived by the subject as new. The solution of a problem is a blending of the application of heuristics and algorithms.

*Illustrative Example*

To illustrate the distinction and blending, consider the "chickens and rabbits problem." The subject is asked to determine how many rabbits and chickens there are if they know that together they have 35 heads and 94 feet.

Carol, an elementary school teacher, read the problem and immediately wrote: $R + C = 35$ and $4R + 2C = 94$. She perceived the problem as being like those she used to solve in algebra class and algorithmically proceeded "without thought." At this point she stopped, unsure of what the next step was. Her algorithmic processing had been broken before resolution of the task, After a few minutes, Carol's eyes suddenly widened and she smiled, "I get it!" she exclaimed. "If all the animals were chickens there would only be 70 legs. But we need 94 legs--so there must be 12 rabbits." to my confused gaze she continued, "Each rabbit has two more legs than a chicken and we have 24 more legs--so there will be 12 rabbits." Heuristics involve the study of the general processes of movement from blockage, although not necessarily towards resolution. A hint which attempts to facilitate these movements is known as a heuristic. In this case, Carol applied a heuristic for which she had not been given, a hint.

Bob, also an elementary school teacher, read the problem and immediately wrote:

$$\begin{matrix} R + C = 35 \\ 4R + 2C = 94 \end{matrix} \Longrightarrow \begin{matrix} 2R + 2C = 70 \\ 4R + 2C = 94 \end{matrix} \Longrightarrow 2R = 24 \Longrightarrow R = 12$$

"There are 12 rabbits and 23 chickens," he asserted 'algorithmically.' When Bob was asked to compare his solution with Carol's, he replied that there was no comparison. They were completely different: he did his algebraically and she just guessed, he indicated. That he used an algorithmic process and she used a heuristic after blockage of algorithmic processing was unclear to him.

*A Schemata for Problem Solving*

In the baseball fable, Nikolai next attempted to find meaningful ways to classify the components of the phenomena under investigation. As a discussion vehicle, the Soviet problem-solving research

has been categorized in the schemata in Figure VII.1. The two aspects, cybernetic and pedagogic, will be considered in the two sections that follow.

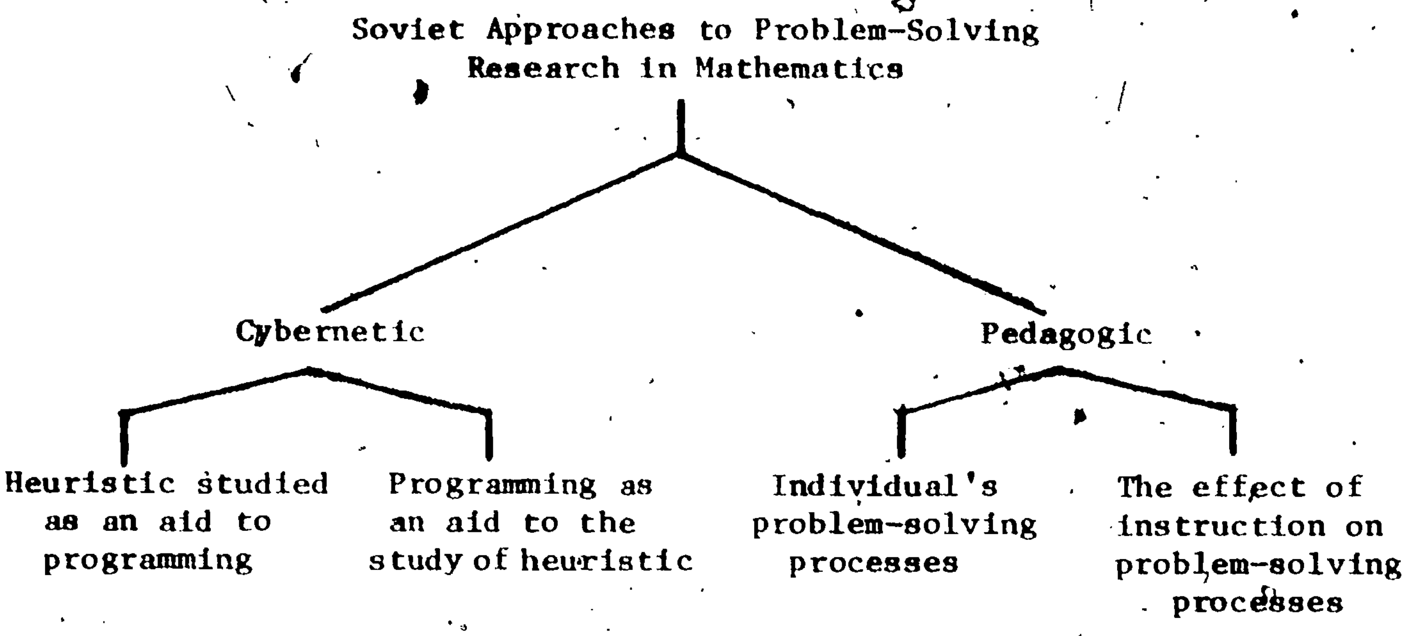


Figure VII.1 A schemata for problem solving.

## B. *Cybernetic Approach*

Among the Soviet approaches to the study of problem solving, there is a growing interest in cybernetics and direct analogies between the work of computers and human thought. Cybernetics, as defined by Rubinstein (1966), is the study of the "determination of processes in the course of which each successive process is conditioned by the results of the preceding one." In terms of our working definitions, Rubinstein has broadened the concept of algorithmic and heuristic activity to include computers in the population of subjects. Algorithmic activity occurs when "conditioning" is reproductive; that is, when the determination of a succeeding process is uniquely determined by its predecessor. Heuristic activity, in contrast, occurs when "conditioning" is productive; that is, when the determination of a succeeding process is not uniquely guaranteed by its predecessor. Through generalization of definitions such as these, Soviet researchers have attempted to form a theoretic foundation for creating direct analogies between the work of computers and human thought.

On the one hand, psychologists such as Tikhomirov seek to investigate human problem-solving processes in order to apply principles used by man to perfect the development of computer programs for the solution of problems. On the other hand, some Soviet psychologists such as Rivkus have utilized research on computer modeling of human behaviour to develop models for investigating the study of heuristics.

Tikhomirov and Poznyanskaya (1966-67) investigated the use of visual search as a means of analyzing heuristics. They explored the application of a system of rules which lead the subjects to a reduction of the search space. Expert problem solvers were found to create what was termed an "orienting zone." The subjects would operate within this zone in a non-linear fashion, leaving and returning to possible solution paths. The "orienting zone" differs from a gestalt in that it is open to an objective analysis by the subject.

In a fascinating follow-up study, Tikhomirov and Terekhov (1967) investigated the use of an "orienting zone" by blind chess players. Here the problem-solving process was largely externalized through tactile activity. Two distinctive forms of planning were noted. When the opponent moved in an expected pattern, the subject would generally react according to a developed plan. After a brief checking of the "orienting zone," a planned move would be made--an algorithmic activity. When the opponent moved in an unexpected fashion, the subject's pattern of checking the board would change greatly. The subject would search for a new plan which would include a new set of hypotheses concerning the opponent's plan. Included in this plan were hypotheses about affective measures. Tikhomirov and Terekhov discussed their findings in terms of a gap between human processing and the heuristic machine.

In a later study, Tikhomirov and Vinogradov (1970) investigated the effects of emotion in the functioning of heuristics. Certain stages of problem solving were found to be capable of identification on the basis of variation in emotion. Here again the focus was on the investigation of human aspects of problem solving to perfect the problem solving of computers.

Parallel to the exploration of heuristic activity of humans as an aid to the development of more refined programming has been the investigation of heuristic programming for its role in the clarification of human problem-solving processes. In a study of the process of solving geometric proofs by computer simulation, Rivkus (1975) examined the four major difficulties encountered in his attempts to simulate students' proof of geometric theories:

1) Intuition rather than logic, found in many students' proofs. Students often relied on inferences made from drawings. These inferences were then used by the students in planning their solution paths.

2) A different classification of ideas as simple or complicated for man and for machine. Some proofs which are easy for humans to solve require sophisticated programming techniques, while others which require few computer steps are relatively difficult for humans.

3) Making creative aspects of proof algorithmic. For example, programming the computer to construct auxiliary lines. It is interesting to note that Rivkus defines "creative" as "that which cannot be programmed."

4) The comparative effectiveness of analytic and synthetic methods of problem solving. The blend of analysis and synthesis used by students did not have sufficient regularity to be used accurately in programming the computer.

In analyzing the results of the experiment, Rivkus calls for further exploration of the heuristic processes of humans in the areas identified via programming difficulties.

### *C. Pedagogic Approach*

North American mathematics educators are perhaps more aware of the Soviet pedagogical approach to problem-solving research than of the cybernetic approach. Since the publication of the fourteen-volume set of translations of Soviet research, Soviet Studies in the Psychology of Learning and Teaching Mathematics, interests in Soviet methodology has grown.

Markova and Abramova (1978) distinguish between two approaches to the psychological study of problem-solving processes, depending on the purpose of investigation. The first approach consists of the study of individual problem-solving processes as the premise of learning, and the second involves the examination of the development of these processes during some specific method of instruction.

A series of eight mini-studies reported by Semenov (1978) is representative of the investigation of individual problem-solving activity. Semenov used normative analysis to demonstrate the psychological reality of several components of the Soviet "theory of activity." In this theory the human problem-solver is viewed as bringing a complex system of operational, object-related, reflective, and personal aspects to a task environment. Two tasks were used in the study: (1) "How many digits are used in paginating a book with 634 pages?" and (2) "To paginate a book, 1,164 separate numerical symbols are required. How many pages are in it?"

In the first mini-study, twenty students were recorded as they thought aloud while solving the *pagination* problem. These solution patterns formed the norm for latter comparison. In the second and third mini-studies, the pagination task was varied for simplification of the numeric operation and clarity of the object-related aspect. The protocols of each of these two groups of twenty subjects were then compared with the first group's protocols. The fourth mini-study provided the baseline for the *pages* problem. The fifth and sixth mini-studies provided information on the reflective component of the cognitive problem-solving activity. Here the norming groups for the *pagination* and *pages* problems were asked to solve the respective problem. The personal individual component of the cognitive problem-solving activity was investigated through a comparison of the norms with the verbal solutions of 43 pairs of students working together. Semenov's use of normative analysis

provides an interesting qualitative analog to quantitative pretest-posttest analysis.

The research of Landa and Krutetskii demonstrates the pedagogical role for Soviet studies of an individual's problem-solving processes. Their research demonstrates how Soviet theory evolves from a study of individual differences to an instructional model. This transition is outlined in the following schemata (Figure VII.2).

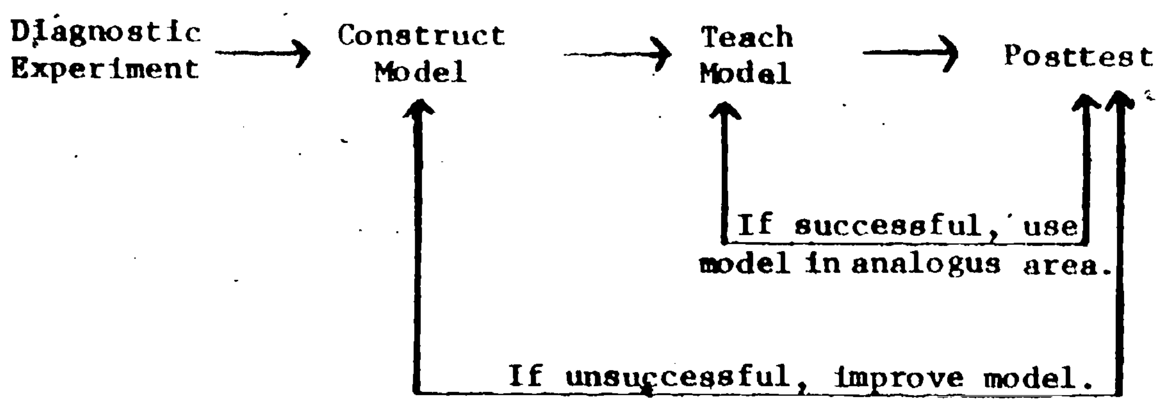


Figure VII.2. Schemata for evolving an instructional model.

For over two decades, Landa (1975, 1976a, b) has directed research aimed at the identification of and instruction in a set of operations "sufficiently general" to aid in the solution of widely different problems. In Landa's investigation into the processes of proof, he sought first to determine the thought-operations involved in carrying out a geometric proof and second to evaluate the effects of teaching these operations to pupils who had previously demonstrated varying degrees of ability to carry out a geometric proof. Students' ill-fated attempts to complete geometric proofs were compared to successful attempts and logically correct proofs. Landa created a set of problem-solving instructions by determining what operations were not performed by the unsuccessful problem solvers. This set was then systematically taught to those students who had demonstrated a lack of problem-solving ability. Posttests administered to these students were used to determine whether the model of operations needed revision or whether it could be tested in some analogous situation in a different area of the curriculum.

In order to trace the thinking processes of students, a diagnostic test on geometric proof was administered to 26 eighth and ninth graders who differed in general ability. The test was divided into two parts. Administered first were ten "moderate to difficult" proofs in which the necessary recall of knowledge was not directly elicited by the diagram and often required supplementary constructions. This set of problems proved to be extremely difficult, with

only 24.4 percent as the average solution rate. Even those students classified by their teacher as excellent solved only an average of 45 percent of the problems. The students fared much better (94.4 percent) on the second set of ten "easy" geometric proofs, in which the diagrams strongly "hinted" at the theorems required for their solution. In an effort to control memory as a variable, the two sets of problems were designed to require recall of the same theorems, axioms, and definitions. Furthermore, the students were directed to use their books or notes at any time they forgot the statement of a particular theorem, axiom, or definition. Three to four problems were administered individually to the students in out-of-the-classroom sessions. During these sessions the students were requested to "think out loud" as they attempted a proof, with the experimenter asking questions about the attempts to solve the problem. Hence, by comparing a student's answers with the observed course of the solution process, the experimenter attempted to trace the thinking process and discover those characteristics of each student's analytic-synthetic operations which were associated with successful problem-solving, as well as those which gave rise to difficulties.

In the second phase of the study, Landa used observed "defects in the cognitive activity of students" (such as not knowing some operations to try, a lack of skill with some of those operations known, an insufficient level of generality of operations, a chaotic process for attacking a proof, and an ignorance of processes necessary for proof or even of what was being done) to construct a model of the processes that should be going on "in the head" of each student. The model was then formulated into a set of instructions:

1) Separate what is given from what is to be proven.

2) Draw the most immediate conclusion from what is given.

3) Go to what is to be proven and determine which of its attributes are already given.

4) Recall the sufficient attributes of the given figure and choose one of these to use as a basis for proof.

5) Isolate the elements of the figure, imagining each element as a part of different figures.

6) Draw any needed elements and let these serve as an aid in analyzing the proof.

7) If these new elements may be the parts of other figures whose properties are necessary to the proof, draw the figures.

8) Make all conclusions possible on the basis of these figures.

9) Re-read the problem to check givens and what is to be proven.

10) Check the steps for a missed or incomplete application of the operations.

In addition to these heuristics, a system of meta-operations was created. These were designed to clarify the instructional model and aid in its implementation. Some examples of these supplementary instructions are:

1) The rules do not set a rigid sequence of operations.

2) Recall all the attributes of the given figure before beginning the proof.

3) Try to place the element in as many possible figures as you can.

After determining what operations students were not performing and developing a set of instructions to train students in the skill of proof, Landa tested the adequacy of his heuristic model. In this phase, 10 ninth-grade students from the original sample were taught a general introduction on the meaning of proof, followed by a system of lessons on the set of operations. Each operation required about three lesson periods for explanation, illustration, and practice. Fifteen specially designed practice problems were used to link each newly learned operation with the set of operations previously acquired. With a gradually decreasing dependence on rule-related guidance, the students proved these geometric problems.

The criterion measure was the degree of success of the students after instruction on those problems from the first part of the diagnostic test which they had been unable to solve. Care was taken to assure that the practice proofs differed from the posttest items in general formulation, as well as with regard to the geometric figures involved. Although the students solved on the average a respectable 87 percent of the posttest problems, Landa places more emphasis on the qualitative evidence of change in students' thinking processes provided by the protocols of the students' "thinking aloud." On the basis of quantitative and qualitative indications of the suitability of the hypothesized model of thought processes, Landa next attempted to test the applicability of the model in the analogous area of language acquisition.

A second example of the pedagogic role of Soviet studies of individual's problem-solving processes is the research of Krutetskii (1976, 1973, 1969). Krutetskii and his students undertook a series of studies aimed at examining the formation and development of mathematical abilities. In a 1958 comparative study of the attempted paths to resolution of 15 algebra students (who had been grouped by ability) on specially organized sets of mathematical problems, Krutetskii (1969) identified three aptitudes "essential for the

mastery of mathematics." These first foundations for the structure of mathematical problem-solving abilities are: (1) rapid, detailed, sweeping generalizations of mathematical material; (2) "curtailment," the truncation of steps of the reasoning process; and (3) "reversibility" of thought, the process of switching from a direct to a reverse train of thought. (A definition of "direct" and "reverse" thought will be given later, when Krutetskii's three aptitudes are discussed in more detail.)

The ability to generalize mathematical material was characterized by maximal, "on-the-spot" generalizations. Capable pupils would often respond to a first encounter with a new mathematical principle or problem-type with a mathematically refined extension to very diverse examples and problems. For example, after one student's first introduction to the formula for the square of a binomial, he immediately applied the formula to tasks such as finding the product of $(C+D+E)$ and $(C+D+E)$ and finding the square of 51.

Krutetskii identified two distinct varieties of curtailment. The first is an ability to truncate steps in the reasoning process consciously and meaningfully. This ability manifests itself to varying degrees in all subjects after a generalization has developed. The process whereby a child develops the concept of addition of whole numbers is an example of this form of curtailment. Given the task of combining 4 apples and 3 apples, a child might first reply "one, two, three, four, five, six, seven ... seven." At a later stage the child will count on. A typical response would be: "four, five, six, seven ... seven." At a final stage the child will immediately respond "seven." At this final stage the child's response is so quick that it is difficult for him or her to indicate how "seven" was obtained. "I just know it," is a typical reply. The second form of curtailment is closely related to this final stage. The distinction lies in the feeling that even the other stages have been curtailed. The subject has an ability to truncate steps unconsciously, as evidenced by the subject's great difficulty in recreating the missing links. The following interview with Nanette (from unpublished research by Rachlin) will serve as an example of unconscious curtailment. At the time of the interview, Nanette was a college freshman enrolled in a non-credit intermediate algebra course because she had failed an exemption test. Her background included two years of high school algebra, geometry, trigonometry, and a senior honors course on logic and problem solving.

**Experimenter:** How much does a fish weigh, if its tail weighs 4 kg, if its head weighs as much as its tail plus half its body, and if its body weighs as much as its head and tail put together?

**Nanette:** (almost immediately) 32 kg.

**Experimenter:** How did you get 32 kg?

**Nanette:** I don't know, it just popped into my head.

Experimenter: Well, suppose you were trying to explain the problem to another student. Could you show them how to solve it?

Nanette: (After several minutes of unfinished starts.) This is what always happens to me. When I was in high school I would get the right answers, but the teachers would mark me wrong because I couldn't show the work.

After several more minutes Nanette completed her solution to the problem, at which time she explained, "That must be how I got it!"

In his study of mathematical abilities, Krutetskii (1969) identified "the ability to establish 'reversible' (two-way) associations in the process of learning mathematical material" as a third essential aptitude for mastery of mathematics. This concept of reversibility is an extension of Piaget's "operational reversibility" as it appears in the work of Johannot (1974). Johannot, a student of Piaget, adapted Piaget's earlier research to high school mathematics instruction. Johannot identified reversibility as a predecessor to meaningful operations with number. For example, successfully responding to Piaget's conservation tasks is said by Piaget to require the ability mentally to link an operation to its inverse. Piaget's concept of inverse requires a definition: for the operation of spreading some objects farther apart, for example, the <u>inverse operation</u> would be to move the objects back to their previous positions, closer together. In general, if an operation T transforms a state $S_1$ into a new state $S_2$, then the <u>inverse operation</u> $T^{-1}$ transforms state $S_2$ into state $S_1$. (Piaget speaks of "reversibility," but the concept is actually that of mathematical <u>inverse</u>.)

Johannot applied Piaget's extra-curricular studies of reversibility in developing a theory of adolescent stages of reversibility based on tasks involving the meaning of complex fractions and the solution of linear equations. Krutetskii interfaced Johannot's notion of reversibility with his exploration of mathematical ability. Krutetskii found that students who applied a short-form algorithm for multiplying the sum and difference of two terms varied in their ability to recognize and apply successfully the reverse operation of factoring a difference of two square terms. As a final example of reversibility, Rachlin (in an unpublished study) asked a class of intermediate algebra students to: a) write down two binomials, b) multiply the binomials, and c) factor the product obtained. The student responses to the last direction (c) were of three types. Some students stated that the polynomial they obtained could not be factored. The majority factored their polynomial, but as if they had never multiplied the two binomials to obtain the product. The remainder quickly wrote the factors and reacted to the odd nature of the question. Though all students had had experience with multiplying and factoring polynomials, they differed on the degree to which they recognized that one of these processes was the inverse of the other.

Krutetskii (1976) further delineated the structure of mathematical problem-solving abilities. This delineation was founded on the basis of an interface of his earlier study with research conducted between 1959 and 1965. Subjects used in the studies varied in age (from six to seventeen years) and in abilities. Three stages of mental activity in solving a mathematical problem were proposed, with one or more abilities classified at each stage. The first stage (gathering the information needed to solve the problem) requires the subject to have a formalized perception of mathematical materials. In addition to the three abilities identified in Krutetskii's earlier study, the second stage (processing the mathematical information) demands the ability to "think in mathematical symbols," a striving for elegance of solutions and flexibility of thought. Finally, Krutetskii's third stage (retaining information about the solution) is dependent on the ability to remember the mathematical relationships in a problem and the methods of solution.

Further utilizing the "teaching experiment," Krutetskii (1973) explored the problem of overcoming the relative inability of schoolchildren to do mathematics. "At the end of a period of instruction" with sixth and seventh graders, who had been classified as "incapable" of learning mathematics (but at least satisfactorily learned other subjects), Krutetskii found an appreciable development of the mathematical abilities of flexibility of thought, formalized perception, mathematical memory, and generalization.

The interactive and longitudinal qualities of the "teaching experiment" permitted Krutetskii's (1973) formulation of three "levels" of "problem heuristic" instruction in which, one after another, the elements of formulating, posing, and solving problems are transferred, in reverse order, from the teacher to the pupil. Another example of the application of the "teaching experiment" in research on the teaching of heuristics in mathematics is given by Kalmykova (1963, 1975). Included in her study of analysis and synthesis is an observational study of the teaching of Petrova, an elementary school teacher classified as an excellent teacher of problem solving.

Reflecting Soviet research practice, Kalmykova provides a detailed rationale for her theory of classroom instruction. The observations of Petrova's classes over a four-year period of time (Petrova taught the same students each year) provided Kalmykova with an opportunity to investigate how well her theory was reflected in the instruction of a model teacher. Analysis of Petrova's problem-solving instruction indicated an emphasis on: reading the problem, breaking down the text of a problem, differentiating concepts, substantiation, flexible questioning, analyzing errors as a method of solving the problem, alternative solutions, abstraction, schematic notation, graphic analysis, and analogy. Kalmykova's research provides an example of a team approach to the investigation of the classroom phenomenon.

The research of Zankov had perhaps the greatest effect on Soviet instruction in the elementary school. On the basis of over a decade

of research, Zankov (1977) and his associates developed a new experimental didactic system, in which the individual subjects of the curriculum are laced with a new methodology for teaching. The experimental methodology involved systematically varying the conditions under which a phenomenon was observed and investigating its relation to other phenomena in order to reveal the active influence of the factor under study. The study was divided into four stages.

In the first stage, the framework of the new didactic system was created through a "teaching experiment" involving one class over a four-year period of time. By focusing on the work in a single class, it was possible to work out each segment of the teaching process beforehand and later to analyze each lesson.

In the second stage, Zankov expanded the network of experimental classes used in the study. Each year the number of experimental classes grew, as did the grade levels included in the experiment. By the end of the fourth year of the second stage; a total of 371 primary classes was involved. Quarterly workshops and visits by the research staff provided an opportunity for the teachers to be involved actively in the development of the new didactic system.

The third stage of the study involved the compilation of experimental textbooks. One of the products of this stage was the preparation of materials that condensed the former four-year primary program into three years. In the final stage, these materials were field-tested in over 1000 experimental classrooms. It was at this stage that the non-instructional variables of the classroom environment became most evident. Besides differences in school conditions (such as the skill level, experience, and work style of the teacher), wide variations were also noted in conditions in the home environment. Zankov noted that the curricular aspects of the new system won immediate and wide-spread teacher support, but that the new methodology had not been accepted as an integral component of the system.

In the area of problem solving, Zankov's research team explored the way pupils in the experimental and conventional classes assimilated the concept of a "problem." Students were read five assignments with six tasks on each. After each task was read, the students were asked to identify which of the tasks were problems and which were not. (An example of a "non-problem" is: "The teacher had notebooks with ruled paper and notebooks with graph paper. She handed them out to the pupils. How many notebooks did each pupil receive?") A sample from each primary class was interviewed individually to establish the methods of classification.

Further studies conducted by Zankov's team of researchers included an investigation of the ability of pupils in second-grade experimental and conventional classes to set up problems and solve them. Zankov found that among pupils in the experimental classes, the formulation of problems takes on a clearcut structure: "The problem-solving experience they have acquired is flexible and yielding, and adequate for the execution of a new assignment."

D. *What Soviet Research Has to Say to American Researchers*

Soviet research, as exemplified by the preceding studies of mathematical problem-solving processes, are indeed varied in their approach. The message which they carry to North American researchers may be equally as varied. Upon reading a report of Soviet research for the first time, Westerners are immediately struck by the surprising emphasis on discussions of national purpose. To the Soviet researcher, these introductions are more than a token 'playing of the national anthem before a sporting event'. Instead, they are the researcher's method of establishing the philosophical foundation of the reported investigation. To understand Soviet research, one must understand its philosophic and historic foundations.

As the American research methodology in instructional psychology was evolving from an emphasis on mental testing, the methodology of the Soviet researcher was evolving from a political and social condemnation of these psychological tests. With the issuance of the 1936 resolution "On Pedagogical Distortions in the Commissariat of Education" by the Central Committee of the Communist Party, a new philosophy of an interrelationship between psychology and education was developed. The educational environment of the child was established as the primary ingredient for success in any given subject area (Menchinskaya, 1969).

Rudik, as reported by Brozek (1963), identified the following six principles of the Soviet psychology:

1) The principle of materialist monism. Mental phenomena (the psyche) are a property of the brain. The psychological processes are regarded as superimposed upon the physiological processes.

2) The principle of determinism. Mental phenomena are the results of interaction between the processes of higher nervous activity and the external environment. [S. L. Rubinstein (1966) refers to the "dialectical-materialist" conception of determinism as the single proposition: "external causes act through internal conditions." To the Soviet psychologist "it is precisely the relationship between the external and internal connections that form the basis of all phenomena, including mental phenomena." It is this connection that ties Pavlov's theory of the external relations of organisms to mental phenomena.]

3) The principle of reflection. Consciousness (mind, psyche) is a subjective reflection of an objective reality. Mental processes are not regulated by special, immanent laws, but have their roots in objective existence. ["It is not thinking that thinks, but man, and it is man and not his thinking that is the subject of thinking (Rubinstein, 1966, p. 63)."]

4) The principles of the unity of consciousness and activity. Man's mind is not only manifested in activity but it is formed in the process of activity. As a corollary mental processes cannot be studied abstractly but only in connection with concrete forms of activity. [Hence Soviet teaching experiments attempt to catch processes as they develop.]

5) The principle of historism. Mind (psyche, consciousness) develops in the process of the historical development of man. Consequently psychologists must study mental phenomena in reference to their genesis and bring out the social roots of different aspects of man's consciousness and personality.

6) The principle of the unity of theory and practice. Through their research, psychologists must take an active part in the solution of practical problems met in the building of socialism, such as development of more effective methods for the training and education of the coming generation.

Searching for a new method of research, Soviet educational psychologists adapted the "natural experiment" of Vygotski. Questioning Piaget's position on the independence of the process of development from the process of learning, Vygotski (1963) theorized a "zone of potential development," which recognizes that "what a child is capable of doing when helped by others" serves as a precedent for "what he is capable of doing on his own." If we fail to recognize this zone of potential development, according to Vygotski, then instruction will trail behind development rather than coax it along. In this light, Vygotski saw the Genevan clinical approach as a static, possibly harmful, method of studying the development of the child, lacking the essential ingredient of experimenter intervention (instruction). From the Soviet philosophical foundation it is easy to see why Vygotski would feel that the chief limitation of Genevan research lies in its inability to explain the transitions from one stage of intellectual development to the next. The question of interest to Soviet psychologists would be: "Why does the child proceed from the preoperational stage to the concrete operational stage and then to the formal operational stage?" (El'Konin, 1972).

Since the Soviets and the Genevans have different research questions, it is not surprising that their research methodologies also differ. The Soviet clinical methodology can be characterized as a loosely scripted one-on-one audio-taped interview. The subject is asked to "think aloud" as he or she solves a large number of curriculum related tasks. As a measure of the zone of potential (next, proximal) development, a series of hints of increasing specificity may be provided. The Soviet clinical study is also often longitudinal in nature, in an effort to catch the development of mental process.

The Genevan clinical methodology is generally a more tightly scripted one-on-one interview. The subject is given a task designed

to be extra-curricular. The intent is to assure the researcher that instruction will not be a confounding variable in the analysis. Often very few tasks are given and no hints are provided. The subjects are asked to retrospect on their solutions and provide reasons for their actions.

The Soviet methodology is particularly exciting for researchers interested in classroom phenomena. In addition to a dynamic curriculum-related clinical methodology, the Soviets have developed the "teaching experiment." As in Vygotski's "natural experiment," the evolving "teaching experiment" attempted to model processes as they developed in the classroom and to organize instruction to influence these processes optimally. Some other general characteristics of the "teaching experiment" which have been identified by Kantowski (1978) are: the subjective analysis of qualitative data obtained in a clinical-interaction setting by recording verbal protocols, over an extended period of time; the planning of instruction in the light of observations made during the previous session; extensive cooperation among classroom teachers and researchers in categorizing and selecting students and designing lessons; and the use of small samples with probing interviews and exchanges with individual students to support the generalizations of results.

Recent Soviet investigations, such as that by Semenov (1978), have explored the affective component of children's ability to solve problems. Thinking-aloud protocols are rich with affective data. In a recent study, Menchinskaya and co-workers (reported in Davydov, 1978) investigated the factors affecting school failures. Failure which traditionally was explained in Soviet research on the basis of poor teaching or poorly designed curricula are now being explored on the basis of personal affective measures (e.g., a lack of motivation to learn, a low opinion of oneself, a lack of confidence in specific abilities, and/or delayed development of various intellectual abilities). Menchinskaya found these factors to be particularly evident during problem solving as opposed to performing routine tasks.

### E. *Limitations of the Use of Soviet Research*

Although interest in phenomenological research is on the rise in the Western world, it is still difficult to obtain current research reports from the Soviet Union. It is hoped that this limitation will wane as the cost-benefit ratio swings in favor of funding for translations and as more students learn to read Russian.

But, at the present time, the use of Soviet research is hampered by the fact that there is very little current mathematics education research available in English. The research reports that do become available are often outdated before they are even translated. The Krutetskii text, published in English in 1976, a decade after it was published in the Soviet Union, has clearly had a great impact on current thinking among North American mathematics educators. But Krutetskii's work did not end in 1966. In fact, Krutetskii published

a new text for teachers a year before his earlier book was available in the West. Had it not been for an article in the journal of translations, Soviet Education, in 1973, the Western world would probably still not know that Krutetskii followed up his work on a model of individual mathematical abilities with teaching studies aimed at improving children's abilities.

It is interesting to note that the first Krutetskii text has probably received more acclaim in this country than in the Soviet Union. (Other translations, such as those of Landa's work, have also sparked interest in North America.) Since the publication of the Krutetskii text, Krutetskii has been viewed by North Americans as one of the leading educational psychologists in the Soviet Union. Yet, when Romberg (1979) recently visited Moscow, he found that many psychologists there were perplexed by our selection of Soviet research for translation into English. Perhaps this reflects a closer philosophical position between Western researchers and Krutetskii's work than exists between Krutetskii and his fellow Soviet researchers. More likely, the selection process of determining whose work should be available in translation is inevitably based on individual judgments and is necessarily open to disagreements.

Unfortunately, there are very few North American mathematics educators fluent in Russian. Nor are there many Russian translators fluent in mathematics education. Hence, the choice of which publications to translate is a difficult one. At present selected passages are translated to pass on to others for review, but someone must decide which passages to translate. Somewhat related to the translation problem is the one of interpretation. The Soviet reporting style is rich in rationale and analysis, but weak in providing a statement of the methodology which would enable or facilitate replication. Much could be gained by having North American mathematics educators participating in exchange programs to study the Soviet methodology.

### F. *Summary of Advantages and Opportunities*

Investigating Soviet research is like looking into an orange crate, rather than the famed "black box." At first glance you don't see anything but the crate, the outer cover. You know that it exists and assume there must be something inside. Then, on closer inspection, you find that you can indeed see pieces of the contents--pieces that entice you and make you anxious to see further. But no matter how you 'stretch your eyes,' you can't see the entire contents. Maybe it should be enough that what you are able to see sparks new ways of looking at your own world. Yet, there is always the anxiety that comes from realizing that unless you can get inside you'll never know what lies behind the slats.

As it was for Nikolai in the baseball story, it is difficult after a brief introduction to the Soviet research to determine what to focus on. Our philosophies are indeed different, but our problems

are very much alike. If the issues of the relative merits of quantitative vs. qualitative research are made focal, much will be lost in the translation. Before such external judgments of right vs. wrong in methodology are made, Western researchers would do well to experience qualitative research. A Soviet-style teaching experiment is not a spectator sport. Soviet research can provide a vehicle for helping Western researchers break away from their mind-set of how phenomena <u>must</u> be investigated.

> The task...is not so much to state a single solution to the large problem that will be suitable for all countries but primarily to demonstrate with patience and all possible objectivity that different approaches to the problem and different solutions of it have a right to exist. In our time there is nothing occurring in the field of education similar to what occurred in mathematics of the last century when everyone gradually realized that it is impossible to attempt to refute every non-Euclidean geometric system and that science is enriched if the notion of "geometry" in the singular is replaced by geometries in the plural. However, as distinct from strictly fixed geometries, such as those of Euclid, Lobachevski-Bolyai, or Riemann, which break down if the nature of their underlying axioms is changed, the premises and postulates which form the basis of education of a certain country or group of countries evolve and can be changed, thus producing progress in the school system. And one effective means for such evolution is international exchange of experience. This is why such exchange has important practical as well as theoretical value and why it is necessary, in discussing even such a special question as problems of mathematics education, to become acquainted with those general conditions under which this question is solved in various countries [from an address to the plenary session of the First International Congress on Mathematics Teaching, 1969 (Markushevich, 1971)].

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# Chapter VIII

## Another View of

## The Value of Studying Mathematics Education Research and Development in the Soviet Union

Mary Grace Kantowski

### A. *Introduction*

Research and development are, by their very nature, continually changing. The knowledge generated by research gives rise to new questions to be studied, new instructional techniques to be tried, and curriculum changes to be made. Two decades ago researchers felt hampered by their inability to deal with quantitative data. The advent of the technological era that led to greater accessibility of powerful computers seemed to herald the coming of a panacea for the educational researcher. To deal with increasingly complex data, more sophisticated statistical tests were developed, and the capability of technology to handle such data easily kept pace. As a result of this phenomenon, a great deal of new information has been uncovered. It is becoming increasingly clear, however, that the sophisticated statistical tests were developed, and the capability of technology to handle such data easily kept pace. As a result of this phenomenon, a great deal of new information has been uncovered. It is becoming increasingly clear, however, that the sophisticated statistical techniques for which the calculations could be ground out by the computer in seconds could not lead researchers into the promised land. Recently the process of evolution of research in mathematics education has given rise to more than a new body of quantitative information about the teaching-learning interaction. Some of the more probing questions spawned by recent studies clearly point to a need for a new methodology that would deal with many of the questions of "how" and "why" that cannot be answered by tests of significance, a methodology that would deal with qualitative data.

Another, related need has emerged. Although there has been a proliferation of research in mathematics education during the last two decades, many classroom teachers fail to see the applicability of the results to their real-life school situations, even when statistical significance is found. Many research studies deal with one idea in a very carefully controlled environment that appears to the classroom teacher to be highly unrealistic. Undeniably, such research is needed and very valuable, but there is also a need to supplement results found in many such short-term, small-scale studies with investigations carried out in actual classroom situations. The real world of the classroom exists with many uncontrolled—and uncontrollable—variables. Surely this real world should be studied, too. The call is for new research procedures that would allow studies to be undertaken in actual school situations.

This need is felt in many areas. Cronbach notes (1975) that "we cannot afford to throw costly data down the drain whenever effects present in the sample fail to reach significance." He adds:

> Instead of making generalization the ruling consideration in our research, I suggest that we reverse our priorities. An observer collecting data in one particular situation is in a position to appraise a practice or proposition in that setting, observing effects in context. In trying to describe and account for what happened, he will give attention to whatever variables were controlled, but he will give equally careful attention to uncontrolled conditions, to personal characteristics, and to events that occurred during treatment and measurement. As he goes from situation to situation, his first task is to describe and interpret the effect anew in each locale, perhaps taking into account factors unique to that locale or series of events. As results accumulate, a person who seeks understanding will do his best to trace how the uncontrolled factors could have caused local departures from the modal effect. (pp. 124 ff.)

These remarks also imply a call for a new research methodology, one sensitive to naturalistic data and to cause-and-effect relationships.

A channel of communication is also greatly needed in mathematics education. In preparing to do a study, a researcher generally searches the literature carefully to find out what has been done and how the proposed study is related to what is known. Unfortunately, due to time lags and other uncontrollable variables, there are communication gaps that result in duplicated work and overlapping studies. A communication network connecting researchers with common interests could alleviate such a problem. A collaborative effort in which a series of related studies are undertaken would also help to bridge the communication gap among researchers and curriculum developers in mathematics education.

Finally, there is a need in mathematics education to integrate the findings from related fields with research in mathematics learning. This is especially true in the field of psychology where, for example, there are many developmental studies and studies related to spatial ability that would be of interest to mathematics educators.

Clearly, many research and development needs in mathematics education exist in the U.S. The needs described above may be roughly categorized into the <u>functional</u>, those related to methodology and other technical and organizational aspects of research and development; and the <u>substantive</u>, those dealing with content and instruction specific to the learning of mathematics itself. Among the most pressing functional needs are means to analyze and synthesize qualitative data, methods of integrating the findings of research and development with actual classroom practice, and viable channels of communication and collaboration among mathematics educators and

between mathematics educators and experts in other fields. Questions related to the substance of mathematics include the study of mental processes, and questions on the development of spatial ability and the relationship of spatial ability to mathematics learning.

It is not unusual that a completely different point of view will often provide insights into a problem and promote new ideas on ways to approach a given situation. Although research and development practices in the Soviet Union may not be completely applicable to our needs, the Soviet work in these areas is so rich with ideas and concepts that are new to us that an in-depth study of their work can not help but add a new dimension to our thinking in these areas, and perhaps point to some new directions.

## B. *Soviet Practices Related to Our R & D Needs*

*Research Methodology*

In studying Soviet research literature, one is struck by two characteristics--an emphasis on details of the thinking processes of individual students and an emphasis on research conducted in actual classroom situations. The Soviet educational research methodology was shaped by Lev Semyonovich Vygotsky, whose influence is still evident in the work of the Soviets today. Vygotsky believed that mental ability is not inherited but formed, and that the development of mental abilities is totally dependent on instruction; if a child did not learn, it was the fault of poor instruction. This premise led to the integration of psychological and educational research at the various Pedagogical Institutes in the Soviet Union, where the primary objective of all research was seen to be the improvement of instruction. Thus, the emphasis in research is on teaching methods that are most effective in producing independent thinking at a high cognitive level.

Vygotsky proposed what he called "genetic" research methodology that would deal with qualitative rather than quantitative data. He was particularly interested in observing thinking processes as they developed and in trying to determine the instructional techniques that were most effective in promoting learning. The Soviets characterize this form of research as dynamic, since it is designed to deal with movement--movement from ignorance to knowledge, from the level of a simple operation to a more complex, from a problem to a solution. The methodology is designed to "catch" processes in their development during this movement.

Vygotsky's emphasis on process continues to influence research and development in the Soviet Union. His individual experiments evolved into the "natural experiment" which, according to Kalmykova (1967), was first described by Lazurskii. A natural experiment (as opposed to the laboratory experiment) is undertaken in a school setting and uses as its content material from the school curriculum. Kalmykova admits that much of the validity of the natural experiment

depends on the skill of the creators of the methods used, as well as the peculiarity of the processes studied. She notes that a great deal of attention is paid to standardizing instruction and clarifying the conditions of the experiments, but admits to some measure of subjectivity and lack of control. She argues, however, that the trade-off is worth being able to operate in the real world. She describes two forms of the natural experiment—the *ascertaining experiment* and the *teaching experiment*.

*The Ascertaining Experiment*

Although the teaching experiment is a methodology that has evoked more discussion in the United States, the ascertaining experiment has just as much potential for application here. As the name implies, the purpose of the ascertaining experiment is to examine a knowledge state or pattern of behavior as it exists in some population. It is most often undertaken to gather information about the status of a skill or ability or to determine error patterns as a prelude to diagnosing reasons for such errors. The clinical methodology is generally employed. Data are collected by having individual students think aloud while doing mathematics; at times students are also asked to examine their work in retrospect and to explain the procedures used. Results are reported by noting generalizations formed on the basis of qualitative analysis (often professional judgments) of the data. Kalmykova (1967) points out that the ascertaining experiment does not allow experimenters to make judgments regarding the acquisition of knowledge, but to "determine the peculiarities of assimilation and application of knowledge and of the formation of habits and of the modes of intellectual activity under given, already formed conditions of learning" (p. 14). Perhaps a few examples would clarify the notion. One of the earliest ascertaining studies, a classic one, was conducted by N. A. Menchinskaya (1969a). Disturbed by the formalism in mathematics instruction and the lack of creative thinking in problem solving among students, Menchinskaya undertook a study to trace the thinking processes during problem solving that could lead her to suggest improvements in techniques for problem-solving instruction. She was interested in how a problem was recognized and understood, how problems were approached (e.g., verbally or visually), and the relationship between productive and reproductive thinking. She looked at problem solutions by subjects from first grade through adult; by subjects judged to be poor, average, and good; and by subjects who had some degree of organic brain damage.

In response to her first inquiry (how a problem was viewed, recognized or understood), Menchinskaya was able to isolate four modes of operating which she saw as four developmental stages or levels of performance. Subjects who performed at the first stage viewed a problem solely in terms of numerical data; in the second stage, the problem was not understood as a whole, so that each part was approached separately and not related to the whole; in the third

stage, subjects again looked at parts of a problem, but then related the parts to the entire problem. Only in the fourth stage did subjects see the problem as an integrated whole and approach it that way. Related to her second question, Menchinskaya found that weaker students preferred working with manipulative materials, whereas stronger students acted symbolically. She noted interesting spatial schemes of solutions as well as verbal ones and found that some students had difficulty verbalizing images and thought processes. Finally, she found that the use of productive or reproductive thought was related to experience; and that problem solving began with reproductive thought, then, if pursued, took on a more productive character.

From her study, Menchinskaya concluded that the type of instruction used in schools needlessly detained students at lower stages of problem solving by forcing them into artificial analysis that hampered creative thought. She suggested that the study of devices used by adults and more experienced problem solvers could give valuable direction for instruction. Broad suggestions for instruction included the need to emphasize comprehension of the whole of a complex problem, to teach various modes of operation, and to make pupils aware of their own thinking processes.

Menchinskaya's vague results, sweeping generalizations, and broad suggestions for instruction could be viewed critically by some Western researchers. Although some degree of criticism might be warranted, it should be noted that her findings were not meant to be accepted as firm conclusions to be implemented in the classroom immediately. Her suggestions were used to form hypotheses for later studies: thus Yaroshchuk (1969) studied the stages of problem solving identified by Menchinskaya in greater detail with larger samples and analyzed his data quantitatively as well as qualitatively, and Gurova (1969), among others, did further study of the effects on problem solving of pupils' awareness of their own mental processes. The suggestions for improvement in instruction served as a foundation for a teaching experiment related to problem solving (Kalmykova, 1975).

Artemov (1975) conducted an ascertaining experiment to examine the use of auxiliary lines in solving geometry problems. He was interested in determining the basis of students' construction of auxiliary lines, again with a view to proposing instructional techniques that might improve student performances. He found that an overwhelming number of students made blind, random constructions, and that the few students who made productive constructions did so on the basis of an operational proposition (that is, a proposition related to some definite goal-orientation; see Talyzina, 1970). He concluded that the obstacle to the construction of auxiliary lines that aided the solution was the lack of a store of operational propositions. He then followed this study with a teaching experiment in which he tested his hypothesis with a small number of students.

Some very interesting ascertaining experiments have been conducted in the Soviet Auxiliary Schools (schools for retarded children). In one such study, Kuz'mitskaya (1975) tried to

pinpoint the source of such children's difficulty in completing simple mathematics exercises. First she studied how students reproduced the statement of the problem, and identified four types of reproduction: 1) fundamentally correct; 2) structure correct, data changed; 3) basic structure changed, only individual components retained, data usually altered; and 4) no reproduction (refusal). In an example of the third type, the problem read:

> There were 19 plain pencils and 6 more colored than plain. How many colored pencils were there?

Pupils reproduced the problem to read:

> There were 20 plain pencils and 10 fewer colored ones. How many colored ones were there?

or

> There were 19 plain pencils and how many other pencils were there?

or

> There were 18 plain pencils and 10 colored ones. How many colored ones were there?

The data made it clear that much difficulty existed even before mathematical operations were performed and that instruction must speak to that difficulty.

In studying solutions to problems, Kuz'mitskaya noted that the relationship between reproduction and solution was complex, but she did find the following regularities:

1) When the conditions of the problem were reproduced erroneously, the problem was usually solved according to the reproduction.

2) In some cases reproduction was erroneous but the solution was correct, pointing to a discrepancy between what is thought and what is said.

3) Even in cases where a problem was reproduced correctly, it was often solved incorrectly.

Kuz'mitskaya noted that the chief differences between her subjects and children in the public schools was the peculiar dissociation of wording from content; that the second signal system (speech) was somehow isolated from the first (thought). Based on these observations, she made suggestions for instructional experiments.

*The Teaching Experiment*

The ascertaining experiment was not intended to look at the acquisition of knowledge. The kind of natural experiment devised for that purpose is the teaching experiment, introduced by Menchinskaya. The objective of the teaching experiment is to construct methods of instruction in accordance with a hypothesis outlined on the basis of observations and preliminary experiments, and to study the effects of those methods of instruction. The instruction may be preplanned or agreed upon by researchers and teachers during the course of the experiment. Often changes are made as the research continues. In a teaching experiment, instruction may be one-to-one or in a classroom setting. Even in the latter case, which is growing more popular, some data are usually gathered from a sampling of "strong," "average" and "weak" representatives of the group(s) under instruction.[1] The data gathered from these subjects are generally qualitative, obtained in a clinical setting by recording verbal protocols for future analysis. Because the emphasis is on process, it is not uncommon for the experimenters to give hints to the subjects during the testing sessions. In this way the experimenter is able to observe the development of certain processes.

Menchinskaya (1969b) describes two forms the teaching experiment may take. The first is what she calls the experiencing form, in which only one mode of instruction is used and observations are made under this mode of instruction to evaluate its influence on mental processes. A second, less common, form of the teaching experiment is the testing form, in which two or more methods of instruction are compared. This form of the teaching experiment is closest to our experimental study.

The experiencing form of the teaching experiment was used by Krutetskii in his series of studies on mathematical abilities (1965, 1969, 1973, 1976). The investigations were conducted between the years 1955 and 1966 using students from the second through tenth grades. By analyzing solutions of carefully organized sets of problems over extended periods with the same students, Krutetskii was able to delineate characteristics of mathematical ability. The organization of the problems was instructional and, as the problems were solved, mental processes were observed in their development. Problems included those requiring generalizations, algebraic proofs, and those with visual-graphic and oral-logical components, among others. Krutetskii emphasized that although some quantitative data were gathered (the number of problems solved and the time to solution), the dynamic indices, such as progress in qualitative aspects of problem solving, were more valuable than the static, quantitative

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[1] The terms "strong," "average" and "weak" are not defined in the Soviet literature. Most probably the decision about which group in which to place a subject is made subjectively by the experimenter and/or the teacher.

ones. This is reflected in the components Krutetskii enumerated in the structure of mathematical abilities, namely (1) the formalized perception of mathematical material, (2) quick and sweeping generalization of mathematical material, (3) curtailment of thought, (4) flexibility of thought, (5) striving for economy, and (6) a mathematical memory. In his earlier writings, Krutetskii included spatial skills in the structure, but later removed it from the "obligatory" structure.

Krutetskii, a deputy director of the Research Institute of General and Educational Psychology at the U.S.S.R. Academy of Pedagogical Sciences in Moscow is, perhaps, the individual with the most potential for influencing research in mathematics education here. In the introduction to the English translation of Krutetskii's work (1976), Kilpatrick predicts that Krutetskii's impact on mathematics education could equal that of Piaget.

Another excellent example of a teaching experiment done in the experiencing form is the Kalmykova (1975) study. The question Kalmykova was researching involved the components of successful instruction for solving verbal problems. To determine components of such instruction, she observed several teachers in their own classroom situations over an extended period. The students of one of the teachers, Petrova, were consistently more successful in solving problems than students of other teachers. Kalmykova tried to isolate the techniques used by Petrova that were different from those of the other teachers, and thus could account for her students' success. She found that Petrova spent a great deal of time on understanding the problem, on thinking through the solution before beginning to operate, and on allowing the students time to reach a solution independently. Kalmykova then analyzed the protocols of older subjects who were successful problem solvers and isolated several techniques they used. She combined these techniques (which she labeled auxiliary methods) with Petrova's instructional practices to propose an instructional model, which she then tried with a small number of students.

Kalmykova's model has great potential for the development of instructional sequences. It is so simple, yet ingenious: observe behaviors that lead to success and those that do not; isolate differences; propose a model using successful techniques; test the model. As with the other designs used by the Soviets, this model is extremely time-consuming and would require cooperation of teachers and administrators, but it has great possibilities, particularly for finding instructional techniques that would work for populations that traditionally have a difficult time learning mathematics.

An interesting teaching experiment using what Menchinskaya calls the testing mode was also conducted by Kalmykova (1958) with a class of physics students. The methodology used is similar to our Aptitude-Treatment-Interaction design. Kalmykova used two methods of instruction and identified two groups of students. Although there were no

statistically significant results based on quantitative data, Kalmykova concluded that one method was better than the other for the slower students, since findings based on the analysis of protocols of students solving the problems indicated a higher level of cognitive activity in the slower students.

In a series of studies related to geometry, Zykova (1969a, b) used both forms of the natural experiment. Using the ascertaining form, she discovered that sixth graders depend very heavily on visual images in their learning of geometry. She also found, after a long series of protocol analysis, that a good number of misconceptions in geometry arise because concepts in geometry are generally taught using diagrams in "standard form." Thus, students incorrectly generalize, for example, that there are only two diameters of a circle--one horizontal and one vertical; that a right triangle must always be "sitting" on one of its legs; and that the base of an isoceles triangle is always horizontal. Zykova hypothesized that the students would learn the concepts more correctly, and thus become better problem solvers, if diagrams used in instruction were drawn in non-standard form. To test this hypothesis, Zykova used the testing form of the teaching experiment. Again she analyzed protocols of students to whom geometric concepts were presented using nonstandard diagrams, as well as protocols of those taught using standard diagrams. Based on analysis of the data from the protocols, Zykova concluded that her hypothesis was supported.

These summaries are gross oversimplifications of the procedures and findings of the studies described. The true spirit of the research and richness of the findings can be appreciated only if one reads the report of the studies and becomes involved in trying to interpret the meaning of the protocols. One of the difficulties American researchers have in reading a Soviet research report is that it is highly descriptive and includes a good deal of anecdotal evidence open to a variety of interpretations that depend on the experiences and even the biases of the reader.

*Spatial Ability*

A question that has stirred a great deal of interest among mathematics educators recently is that of the relationship of spatial ability to the other mathematical abilities and to achievement in mathematics. In the U.S. spatial ability is generally treated as an aptitude, with the assumption that it remains relatively stable as an individual develops. One of the most fascinating notions seen in the Soviet research on spatial ability is that this ability is not static, but may be improved under instruction. Vladimerskii (1971) has put forth a series of exercises for sixth-grade and ninth-grade students that he claims will improve spatial imagination. The sixth and ninth grades were selected for administration of the exercises because of the relationship of the mathematics curriculum at those grade levels to the content of the exercises. A large segment

of the sixth-grade mathematics syllabus is devoted to the geometry of the plane, while the study of three-space and three-dimensional figures begins in earnest in the ninth grade. Logically, the sixth-grade exercises are designed to improve what Vladimerskii calls spatial imagination in the plane, while the ninth-grade program deals with three-space. Many of the exercises deal with figures in nonstandard positions both in the plane and in three-space which, as Zykova found in the studies cited above, helped students in correctly learning concepts. Other exercises deal with transformations of figures and mentally operating with them. The exercises themselves make an interesting study; the question of if and how spatial imagination can be trained is one that deserves investigation and support.

*The Geometry Curriculum*

In a related area, there is a great deal of discussion in the U.S. mathematics education community concerning the role of geometry in the mathematics curriculum. By and large the study of geometry in the U.S. is concentrated in the traditional formal course in the secondary school. Although some geometry is contained in elementary school textbooks, its actual inclusion in the school curriculum quite often depends on the decision of the individual teacher to do more than just a minimum of work in geometry. Usually a token introduction to some basic definitions and some common theorems is the only geometry covered at the elementary level. In the Soviet Union, on the other hand, the study of geometry is an integral part of the mathematics curriculum from the early years. The Soviets follow the Van Hiele model (Wirszup, 1974) in their geometry sequence. Basic geometric structures are systematically introduced very early in the curriculum. Relationships among structures are introduced later in the elementary schools, as are some of the metric relationships and the applications of some of the common formulae. In the sixth grade, for example, geometry is studied for two hours each week. The work includes constructions of altitudes and medians of triangles and properties of parallelograms and symmetries are studied and the Pythagorean theorem is introduced. In the eighth grade, there is an interesting integration of arithmetic and geometry in the syllabus. Ratio and proportion are introduced in arithmetic; geometry topics include similarity, proportionality, theorems on altitude to the hypotenuse of a right triangle, and theorems related to circles. Sample test questions illustrate how the work is integrated, as well as the level of work expected from the student:

> Sample Problem 1: Construct a right triangle on the basis of a 3:7 ratio of its legs and a given altitude drawn to the hypotenuse.

> Sample Problem 2: A tangent and a secant drawn from the point A to a given circle are equal to 16 cm and 32 cm respectively. The secant is 5 cm from the center. Find the radius of the circle.

Sample Problem 3: The common chord of two intersecting circles is extended. Tangents are drawn to these circles from a point taken on the extension. Show that their segments from the given point are equal.

In the ninth grade, geometry is studied more rigorously. Definitions, theorems, and axioms related to properties of the plane and three-space are introduced. Finally in the tenth grade, more complex theorems related to three-space and polyhedra are studied. Throughout the curriculum, the ability to operate with geometric concepts is given as much emphasis as is computational skill. There are advantages to the early introduction to geometric concepts and to experiences in which manipulative materials are used and formulae representing geometric relationships are used. When students are ready to reason formally, they will have had a good deal of experience with the concepts.

In the U.S., geometry is treated as a deductive system, with emphasis on formal reasoning. Indeed, some of the opponents of leaving geometry in the secondary curriculum argue that the study of logic could accomplish the same goal. Chetverukhin (1971) gives an interesting argument that could refute those who would remove geometry from the curriculum or replace it with logic. He suggests that there are two modes of instruction in geometry, the formal-logical and the visual-applied. He observes quite correctly that the emphasis has been on the former method and that the latter has been neglected. By observing protocols of students learning geometry, however, researchers have gathered evidence that students who complete correct formal proofs may actually be reasoning from the diagram. Chetverukhin concludes that there is a need for emphasis on the active use of the diagram in instruction in geometry.

Many Soviet researchers have studied the influence of the diagram in solving geometry problems. Studies range from Zykova's study (1969a), in which she dealt with very simple concepts in geometry and the false generalizations made by students as a result of a "standard placement" of a diagram, to a series of studies by Artemov (1975), in which he dealt with the importance of the visual-applied aspects of geometry in the solution of very complex geometry problems.

Soviet psychologists have also studied the problem. Zavalishina (1974) looked at the functions of visualization in problem solving. She noted two functions of visualization--the function of fixation of information in which a visual image is used like a word to represent a symbol, and the means of analysis of logical conditions where the drawing is actively used in the solution of the problem. Zavalishina uses examples from the studies of Zykova (1969a, b) and Kabanova-Meller (1971) to show how these functions are exhibited in school mathematics, thus providing another example of the effective communication network among pedagogical and psychological researchers.

Using the ascertaining experiment described above, Talyzina (1970) was able to identify two ways in which propositions and theorems in geometry are used and to relate their use to success in problem solving. By analyzing the recorded verbal protocols of students who were able to solve geometry problems as well as those who had little success, Talyzina identified processes that she called the "use of objective propositions" and the "use of operative propositions." The subjects who simply recognized propositions related to the problem that referred to properties of the figures involved were said to be using "objective propositions." Those who suggested some proposition that indicated what should be done were said to be using "operative propositions." The use of "operative propositions" showed a goal orientation. Talyzina found a high correlation between the use of operative propositions and success in solving geometry problems.

Such studies of differences of processes in successful and unsuccessful problem solutions are needed. Talyzina observed the use of processes in subjects at various levels of sophistication in mathematics (elementary, secondary, college students), and with varying abilities to solve the task problems. Such systematic observation of subjects at many points of the knowledge continuum as well as the ability continuum is a technique often employed by the Soviets that could be useful in our research on the learning of mathematics and on the development of mathematical ability.

What Talyzina describes as the "use of operative propositions" is very similar to what Duncker (1945) describes as "analysis of conflict" and to what is seen in the Information Processing Model of problem solving (Newell and Simon, 1972) as search paths through a problem space. Talyzina's examples from student protocols in which "operative propositions" are used also link them to Polya's planning heuristic (1957). "Objective propositions," on the other hand, seem to be related to Duncker's "analysis of the material" and the Information Processing System operators that produce subexpressions from another single subexpression. Although many of the classical researchers of problem-solving behavior and, more recently, the information processing theorists have studied similar behavior, the Soviets have looked at problem-solving behaviors in the context of the school curriculum. The comparison of Talyzina's work with that of the Western researchers cited again demonstrates the need for communication among disciplines. Most studies of complex problem-solving behavior here have been done by psychologists, generally using "puzzle type" problems. Yet it is continually acknowledged by American mathematics educators that the ability to solve complex nonroutine problems is a goal of mathematics instruction. Research on complex problems of the type used by the Soviets, preferably with the collaboration of psychologists, is clearly needed.

*Curriculum Development*

The Pedagogical Institutes in the Soviet Union provide a mechanism to deal with the problems of communication and collaboration. The Soviets believe that the primary purpose of research is the improvement of instruction, since all intellectual development is preceded by instruction (Zankov, 1977, p. xi). Therefore, virtually all research and curriculum development efforts in the Soviet Union are tied together at one of the nation's many Pedagogical Institutes. Projects often run for years under the direction of a senior researcher working with a staff of colleagues or proteges. One of the largest curriculum development projects ever undertaken in the Soviet Union was headed by L. V. Zankov, beginning in 1957. The first phase of the experiment, the development of the framework for a new didactic system, took four years to complete. This phase used the methodology of the experiment form of the teaching experiment.

In this first phase, each segment of the teaching process was thought out beforehand, carried out, and subsequently analyzed. In addition to studying the cognitive aspects of the curriculum, "The life of the classroom collective as a whole and each school pupil as a unique individual were studied for a period of four years" (Zankov, 1977, p. 39). The experimental classrooms had an adjoining room for the researchers. Video (photographs) and audio (tapes) records were kept. After four years of work on the experimental curriculum, members of the staff reported the findings of the research at a conference of teachers, and nine teachers were selected from among those who volunteered to participate in the second phase of the study, which consisted of an initial implementation. The following year twelve schools were added to the original nine, which continued to participate in the experimental program. After that, sixth-year textbooks were written to prepare the materials for use by larger numbers of classes. The number of participants increased rapidly every year after that. By the 1965/66 school year, eight years after beginning work on the experimental curriculum, there were 1,034 classes participating. All testing and evaluation data from the participating schools were returned to the Laboratory for Problems of Teaching and Development at the Academy of Pedagogical Sciences for analysis.

Although such a tightly controlled model is not possible in our country since we do not have a national curriculum, some aspects of the model could certainly be followed in some of our R & D centers. Whereas the Soviet model includes all aspects of research and development, including formative and summative evaluation of the developed materials as well as the dissemination and implementation of the materials in the classroom, our models have traditionally not included the last phase, since materials developed in projects here are generally turned over to commercial publishers. It would be interesting to see a project in this country, even a very small one, taken through all of its stages from the initial conception of the problem through the research and development phase, into the

implementation phase, and finally through a thorough evaluation under the direction of a research-and-development team under a single director.

Although none of the major projects undertaken in the U.S. is carried through on all aspects of the Soviet model from the conception of the idea to the implementation and evaluation of the materials, several projects have reflected some aspects of the Soviet model. The writers in the Madison Project, for example, worked during the school year and new materials were written as a result of close work with actual classes. If something went well, they would try it in more classes, rewrite, try it again, and so forth. If something was going badly they tried to determine what it was, and to correct it. This procedure was similar to the instructional aspect of the Zankov project. The closest we came to the evaluation of student performance after implementation was in the work of the School Mathematics Study Group (SMSG), which followed through with the NLSMA and ELMA studies. Even the many critics of some aspects of the large curriculum projects would have to agree that each had some positive impact on the study of mathematics in the schools.

### C. *Concluding Notes*

Only a few examples of research and development practices in the Soviet Union in selected areas have been examined, but even these few examples should provide food for reflection and, perhaps, some interesting questions and avenues of research to pursue. Several other constructs that appear quite consistently in the Soviet literature merit study and discussion, if not serious exploration. One of the most interesting is the "zone of proximal development," a concept similar to what we call readiness. The zone is seen as the distance between what a child is able to do independently and what can be done with the aid of some form of instruction. The behavior is studied under a number of cues to see how far the child is from acting independently. The concepts of curtailment [truncation of the reasoning process (Menchinskaya, 1969c)] and valence [focus on essential elements of a concept (Artemov, 1975)] and the device of creating conflict situations (Kalmykova, 1967), should be of great interest to researchers in mathematics education, particularly those interested in processes involved in problem solving.

Even the few examples discussed above suggest that we could reap great benefits from a close study of research and development practices in the Soviet Union. Certainly there are cautions to be exercised. There are basic differences in philosophy that result in basic differences in educational practice. For example, we have no national curriculum, education is left to the individual states and communities; there is much more choice in our electives system, which also results in a variety of mathematics offerings; the mobility of

our lifestyle makes longitudinal studies with small numbers of students much more difficult; our teacher-training programs contain much variety. In spite of the differences, there are many areas where we can learn. The Soviet emphasis on the school curriculum, their communication networks, and their confidence in the integrity of the professional judgments of the researchers could give us new avenues to tread in our search for more effective practices in the mathematics classroom.

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# Chapter IX

## Summary

Robert B. Davis

### A. *Similarity and Difference Are Both Important*

The case for deeper study of Soviet research and development in mathematics education is strong, and can be stated simply: Soviet R and D is often different from typical U.S. work, and Soviet R and D is often of high quality and serious intent. These two facts, together, imply that we can gain substantially from studying Soviet work.

The chapter by Romberg shows clearly the value of new points of view. One need not--and should not--become entrapped in arguments over the superiority of one narrow point of view over some other. What is needed is diversity, openness to high-quality alternatives, the potential inclusion of that which has been excluded. The case has been well-argued by Alfred North Whitehead:

> In the history of education, the most striking phenomenon is that schools of learning, which at one epoch are alive with a ferment of genius, in a succeeding generation exhibit merely pedantry and routine. The reason is that they are overladen with inert ideas. Education with inert ideas is not only useless: it is, above all things, harmful--*corruptio optimi, pessima*. Except at rare intervals of intellectual ferment, education in the past has been radically infected with inert ideas...
>
> ...Every intellectual revolution which has ever stirred humanity into greatness has been a passionate protest against inert ideas. *Then, alas, with pathetic ignorance of human psychology, it has proceeded by some educational scheme to bind humanity afresh with inert ideas of its own fashioning.* [italics added] (Whitehead, 1929, pp. 13-14)

If one believes, as Whitehead did, in the importance of not being bound to a narrow or incomplete world view, then the two facts are enough: Soviet work is good, Soviet work tends to be different, therefore we have something to gain from studying Soviet work.

Besides differences there are also similarities (as, again, Romberg shows clearly). Soviet R and D aims at improving the learning of mathematics by students in the Soviet Union. United States R and D aims at improving the learning of mathematics by students in the United States. These two nations have much in common, as

large modern nations, subjected to the stresses and opportunities of industrialization. Consequently, at the most fundamental levels, the goals of Soviet work cannot be substantially different from the goals of U.S. efforts. There must be considerable similarity between what we need to learn from studying Soviet work, and what we need to learn in general.

Any adequate statement of the case for studying Soviet R and D must reflect these two truths: the importance of our differences, and the fundamental congruence of our basic goals. What, then, are the goals of R and D in America?

### *B. U.S. Needs*

Any knowledgeable person can compose a long list of educational needs in the United States. We avoid a long list because it lacks emphasis and destroys any sense of direction. Like the classified ads, it lacks a good story-line. Consider, instead, the following short list of eight important needs:

1) *The United States needs better conceptualizations for analyzing the processes of mathematical thought.*

2) *The United States needs better arrangements for developing and implementing improvements in mathematics curricula.*

3) *The United States needs a deeper understanding of the motivational issues that play a large role in determining what a student will learn.*

4) *The United States needs better forums for research communications.*

5) *The United States needs better forums for the discussion of value issues and social issues, as they relate to mathematics curricula.*

6) *The United States needs better forums for discussions of the philosophy of research and development in education.*

7) *The United States needs more effective concern for the art of teaching.*

8) *The United States needs better provisions for the continued professional development of teachers.*

Why is this list limited to U.S. needs? Surely any successful program of Soviet-U.S. collaboration must be *mutually* beneficial! Of course that is true, but a reasonable regard for one's mandate and one's competence requires that each nation be allowed to define

its own needs and goals. I suspect that a list of Soviet goals would, *on a fundamental level*, show considerable overlap with the present list, but that, clearly, is for Soviet colleagues to determine for themselves.

*The main argument of this chapter is that the determination of possible gains to be had from communication and cooperation between the U.S. and the U.S.S.R. in the area of mathematics education R and D must be seen as part of the question: WHAT ARE THE BASIC NEEDS OF THE UNITED STATES IN THIS AREA?*

The list of eight needs presented above was heavily influenced by a thoughtful analysis of the situation in the United States, arising from the work of a committee of the National Council of Teachers of Mathematics and reported (excellently!) by James Fey (1979) in the *Mathematics Teacher*. The report draws on three sizeable studies of the mathematics situation in the United States at the present time: Weiss, 1978; Suydam and Osborne, 1977; and Stake and Easley, 1978.

The picture that emerges is one of a national educational effort that shows *important successes* (else how could we have our space effort, our micro-wave ovens, our computers and hand-held calculators and jet airplanes and color TV?), *important failures* (witness high school graduates with no skills that render them employable; witness the few women who study mathematics; witness the mathematical errors that pass (almost) unnoticed in signs, advertisements, and even laws; witness the general popular fear of mathematics), *and no disposition to change*.

Taken together, these reports constitute a "State of the Union" message concerning mathematics education in the United States today. The flavor is best captured by a few quotations:

Statements by mathematics teachers:

> We offer excellent training for the college-bound.... For [all students] we offer the discipline that comes from a rigorous regime of study. (Mary Lee Smith, 1978, p. 11)[1]

> We've found that traditional methods [of instruction] work. This is the way it was taught to us in high school and the way it was taught in college and the way it works for us... I don't think kids can handle inquiry...They just don't have the background or sophistication. (ibid., p. 11)

> The problem most common to us teachers is that we can't keep *real* algebra exciting for the students. When the

[1] In the reference list, the various chapters of the *Case Studies* report can all be found under the listing for Stake and Easley (1978).

students ask, "How am I going to use this stuff?" our usual answer is foggy. The only answer is in higher mathematics." (Denny, 1978, p. 69)

What is lacking in the textbook is applied mathematics. Even the average text goes into properties. A kid just does not have to know 'commutative' and 'distributive' to function in the world. (ibid., p. 68)

Hard and boring. That's why I got into math. Trying to figure out how to make it not boring. I have been disillusioned. It *is* a drag. (ibid., p. 106)

Reading will panic most of our kids in math, even some of the [best]....If two sentences are given to direct students in a mathematics problem, not more than two of thirty can go directly to the work. I don't have time to teach statement problems. We spend too much time on theory and not enough on statement problems in the curriculum. So we are losing a chance to develop a child's reasoning ability. Statement problems are the key. (ibid., p. 71)

Statement from a science teacher:

I can see the results of the new math on all the students I've had here. As sophomores, they don't know how to do long division [and] most don't know how to figure up percentages. It really shocks me...They're lost to science as far as math is concerned. (Peshkin, 1978, p. 9)

Statements from elementary school teachers (who, in the United States, are NOT specialists in teaching mathematics, although they *are* the people by whom mathematics is taught--in addition to their other duties):

Modern mathematics? I dislike it...[The text] shows three ways when one will do. The brass tacks are learning addition and subtraction. That's *it*. (Denny, 1978, p. 31)

This book has too much esoteric garbage in it. It is simply too hard. The geometry is silly [to try and teach] even for our best third graders. So we all skip it. (ibid., p. 33)

We are fortunate not to have gone way out for the new math. We have stuck to the basics throughout it all and the results that are coming in show we were right. (ibid., p. 33)

To mathematics education specialists, these quotations may suggest a weird and disquieting combination of knowledge and

ignorance, of complacency and concern. This same impression is suggested by comments such as the following, the first from a parent, the second from a student:

> My kids understand the basic number system much better than I ever did at their age...They know, for instance, what multiplication *is*, rather than just knowing some tables by heart. In the long run I'm sure this will be a big advantage to them, especially when they come to take algebra or calculus. (Walker, 1978, p. 52)

> Science and algebra and geometry are just teaching you to think, even if you don't understand. It imprints in your mind to use a process, to think for different situations that you come across...Math--I couldn't get by without it. (Peshkin, 1978, p. 33)

Mathematics classes in U.S. schools emerge from these studies as "boring" but "comfortable." Consider these statements from observers:

> In all math classes I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (Welch, 1978, p. 6)

> A general comment about math classes is that they were dull. Science was perceived as being more fun...I got the impression many students looked forward to science but no one seemed to look forward to math. (ibid., p. 29)

As Fey points out, what goes on in U.S. classrooms has very little in common with the recommendations of the "new mathematics" projects, or of the college professors who recommend "inquiry-based teaching" or "discovery learning" or "laboratory exploration" or "individualized instruction." Is it possible that the "experts" are wrong? Are there realities of classroom life that constrict the range of feasible approaches? Welch (1978) reports:

> Although it seemed boring to me, students and teachers seemed comfortable with it. Apparently it fulfills student expectations and provides the students opportunity for closure. (p. 6)

Fey cites a comment by Hoetker and Ahlbrand (1969):

> If the recitation is a poor pedagogical method, as most teacher educators long have believed, why have they not been able to deter teachers from using it?
>
> Or, is it not possible that the practicing teachers are right and the professors unrealistic, and the recitation --for some reason--is the best pedagogical method? Or the only practicable one for most teachers. (Fey, 1979, p. 495)

Intellectual history should teach us not to ignore the wisdom of competent experienced practitioners. But, quite aside from practitioner wisdom, there is a question of national needs in economic and cultural terms, and in terms of the kind of world that our teachers live in, and that our young people are taught to expect. Is it in fact the case that those who are complacent are correct? Is it perhaps true that change is *not* needed? The overall impact of the studies does not suggest an affirmative answer. As Fey summarizes the situation:

> One could comfortably accept this latter explanation of predominant teaching styles and conclude that teaching, as it is now commonly practiced in mathematics classes, is about as effective as one can reasonably expect. However, the case studies and the survey produced frequent comments that students find the study of mathematics boring and teachers find motivation of students to learn mathematics one of the most difficult problems they face. When teachers were asked what aspects of their jobs they most needed help with, *at all levels they mentioned learning new teaching methods and implementing discovery/inquiry approaches.* [italics added] (Fey, 1979, pp. 495–496)

There is abundant evidence that all is not well; consider these statements by teachers:

> ...I've had a lot of spark taken out of me in the last two years. We hear administrators talking about meeting the needs of students--individualization. But we never get time off to develop these things or the financial support...I've talked to them about getting student-teacher relationship. But I find it very difficult to stand up and play Johnny Carson every day. The kids don't want to hear lectures, they want to *do* things. I always thought that the main goal of education was teaching kids, now I find out that the main goal is management. (Mary Lee Smith, 1978, p. 9)
>
> There is abundant evidence to show that we are encouraging superficial learning in some of our [best students].

> Sure, they do well on the tests. Our materials on hand encourage this. The algebra book, for instance, is pure abstraction. The really good memorizer can go right through and not really have it at all. (Denny, 1978, p. 68)

> We need to be working with teachers, not checking on them...Education is generally a negative enterprise toward children, toward teachers. It is a highly structured reward structure which emphasizes the negative. Those who get rewarded are those who make the fewest mistakes. (Denny, 1978, p. 8)

> There is no way to motivate a certain number of kids. They put forth absolutely no effort. I will not be held accountable for teaching an unteachable student ....Why is accountability the teacher's responsibility? Why is it always my job to solve every problem a child manifests in school? I am not in charge of the math program. So how can I be accountable for it? (ibid., p. 8)

Mathematics teachers painted a picture of "going stale, of burning out" (Fey, 1979). Louis Smith (1978) describes the condition as:

> A flatness, a lack of vitality, a seeming lack of interest in the curriculum by both the teacher and the children, a lack of creativity and curricular risk taking, a negativism toward the children--they're spoiled, they don't care, they don't try--and sometimes a negativism toward colleagues, administrators, and college and university training programs (often decades ago). (p. 84)

What do teachers think that mathematics really is? Fey (1979) reports:

> Several case study observers asked teachers to explain what they saw as the big ideas in mathematics. From elementary teachers the most common response was that mathematics is a collection of rules and procedures to be learned to a level of near mechanical proficiency. (p. 498)

From students (unsurprisingly) one gets a similar view:

> ...students seem to expect emphasis on facts and memorization in mathematics, along with the neat closure that comes from a discipline with well-defined procedures and "right" answers. In one science class a site visitor suggested to students that they might find D'Arcy Thompson's book on *Growth and Form* an

> interesting guide to modeling of scientific observations by mathematics. The observer commented, however, that it was clear from the student responses that this was not regarded as mathematics. The same narrow conception of disciplines seems to plague science as well. (Fey, 1979, p. 498)

How well has R & D been helping teachers and students? Not very well at all, according to present evidence. Suydam and Osborne (1977) reviewed the seemingly relevant R and D literature, and their review has been summarized succinctly by Fey:

> ...recent research and expository literature offers little immediate hope that instructional research will yield new ideas or convincing support for any existing strategy or technique. There is no consistent pattern of results favoring recitation, discovery, small group, or individualized approaches in mathematics teaching; there is no demonstrably superior way to identify the knowledge, experiences, or personal traits of people who will be consistently effective teachers. (Fey, 1979, p. 496)

When you go to the supermarket, you had better have along a thoughtful shopping list, prepared carefully beforehand. It is in this spirit that, before considering Soviet possibilities, we have tried to ask: *what is it that the United States needs?*

### *C. The "Flavor" of Soviet Work*

To put the matter *very* briefly--leaving the earlier chapters to speak for themselves on matters of detail--one might say that, speaking very generally and broadly, Soviet R and D work shows common sense in four important respects, and might be labelled "dynamic" in four different senses. (It should be pointed out that we use "common sense" here as a term of approbation. There is enough general wisdom among mathematicians, teachers, and others who work with mathematics that it is *good* for things to look effective, appropriate, and useful.)

1) Soviet R and D tends to appear *to mathematicians* as sensible inquiries, dealing with the kinds of things that one cares about. The mathematical content--proofs and constructions in geometry (as in the work of Artemov, 1975), algebraic proofs (Krutetskii), problems in visualization (Krutetskii, Zykova, Zavalishina, and Vladimirskii, among others), ratio and proportion, creative problem solving (Talyzina, Menchinskaya, Artemov, Krutetskii, and others[2])--appears to mathematicians to make sense; this sort of thing is part of the genuine content of real mathematics. The *kind of thought*

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[2] Some publications by these authors are included on the list of references.

*process that is studied* is recognizable to a mathematician; this is in the right ball park in terms of how competent people do think about mathematics (cf., e.g., Krutetskii, 1976). The *kinds of questions that are asked* will be of recognizable interest in a mathematician's view of things.

2) Soviet R and D in mathematics tends to be sensible *in the eyes of teachers*. When, for example, Kalmykova found that Petrova was an unusually effective teacher, she set out to determine what it was that Petrova was doing that tended to produce such good results. This kind of question is of interest to all serious teachers, and the kind of answers that Kalmykova reported make sense to teachers.

3) Soviet R and D in mathematics education tends to look sensible to the *users* of mathematics; it includes an adequate concern for practical problems at all levels of sophistication.

4) To an outsider, it would seem that Soviet R and D is sensibly directed toward the national economic needs of the U.S.S.R. Finding themselves short of competent computer scientists, the Soviets have created special high schools for prospective computer science specialists--the U.S. did this, years ago, especially in New York City (as in the case of the Bronx High School of Science), but in recent years this seems to have been neglected.

(There are, of course, some exceptions. It does seem that the present school program in the U.S.S.R. is very formal, making too little contact with "shop mathematics" to meet the needs of future machinists and draftsmen; if this is true, it seems likely that attention will be addressed to this issue before much longer. And the Soviet shortage of technically trained people seems very persistent, which necessarily raises questions about how "well-tuned" the Soviet educational programs actually are.)

So, in at least these four ways--and one can think of others--Soviet work seems for the most part to be "sensible" and "reasonable." One never gets the feeling that the work is foolish or meaningless.

Soviet work is also *dynamic*, again in several different meanings of the word:

1) It is dynamic in the sense that it typically deals with a student's thought processes, and not merely his or her answers. Krutetskii, among others, makes this abundantly clear; indeed, this appears to be a persistent theme throughout Soviet R and D activities.

2) It is dynamic in its interest in the "zone of proximal development"--that is to say, *not* just what is in a child's repertoire of reliable knowledge and technique, but also what the child is ready to move on to, to master, to carry out with

a minimum of prompts and help from adults. To many Americans, as well as Russians, this is often the attribute of a student that is of the greatest relevance. For a student beginning calculus, we care especially that the student is in a position to be able to learn calculus effectively. This is NOT the same thing as the statement that the student possesses--perhaps in what Whitehead might call our "inert" form--previous knowledge of, say, the half-angle formulas in trigonometry. Many mathematics courses list as their prerequisite "adequate mathematical maturity," and this may well be a correct statement of the true prerequisite. One can know the half-angle formulas, yet still lack "adequate mathematical maturity" to undertake the study of calculus, and *vice versa*, many students who *do* possess "adequate mathematical maturity" for beginning the study of calculus do not know the half-angle formulas (but, of course, if in fact they *do* possess "adequate mathematical maturity", they can easily learn these formulas). A static listing of some of the more obvious items that sit on the shelves of a student's intellectual warehouse is not equivalent to an examination into what the student can *learn* and what the student can *do*. It is this latter question which is more important.

3) Soviet studies are often dynamic in the sense that they follow the same students for as long as four years, thus dealing with the kind of significant growth which is of the greatest interest for serious educational programs. (Here, too, one could cite remarks of Alfred North Whitehead.)

4) Soviet R and D work is also dynamic in the sense that it sees education as an *urging forward* of the child's intellectual growth, a deliberate effort to move the child up to higher levels of functioning. It is in this sense that Soviets tend to sound hostile to Piaget: they find too much in the Piagetian school that seems passive to Russian eyes, resigned to an inevitable pace of developmental growth, with neither the responsibility, nor the desire, nor the possibility of moving the child forward by deliberate *teaching* acts. (Piaget is probably not personally guilty of such passive resignation, and his own childhood hardly displayed it, but it is an undeniable truth that many self-proclaimed "Piagetians" do hold such views.)

The view that more ambitious school programs are futile is widely held by American teachers. Consider, for example, the following remarks by U.S. teachers:

> No algebra should be taught in junior high. Fortunately, nature is on my side and very little algebra *can* be taught in junior high. (Denny, 1979, p. 102)

> I wish they were almost taught *no* theory down there. Back to basics. That means delaying teaching of theory. (ibid., p. 102)

These are views the Soviets would disagree with.

The issue of *teaching more mathematics to children* is one that deserves special attention. Krutetskii, Zankov, and Davydov (cf., e.g., Davydov, 1975) all report, from Russian experience, that children can, indeed, learn considerably more mathematics than the traditional curriculum attempts to teach--although special methods of teaching may be required to achieve such goals. These Soviet reports appear to match very closely a large number of U.S. studies that have come to the same conclusion, including work by Johntz, Kaufman, Page and the present author. Given the consistency of this finding, and the reluctance of U.S. teachers to believe it and to act on it, this would seem to be an area where the careful combining of U.S. and Soviet experience could help to resolve what, at present, is something between a mystery and a dilemma.

### D. *Some Specific Areas for Soviet-U.S. Cooperation and Communication*

1) We have just seen one such area--*can* children learn more mathematics, and, if so, why are schools and teachers so reluctant to attempt it?

What other areas of potential cooperation come strongly to mind?

2) One, surely, is *the study of the process of mathematical thought, and in particular the process of solving mathematical problems*. This has been a matter of concern for decades, if not centuries. One relatively early study, by Bloom and Broder (1950), describes the need as it was seen by those authors at that time:

> Mental processes represent a very difficult and complex subject for study. To a large extent, we have been limited in this study by the wide-spread emphasis on overt behavior as the major acceptable type of evidence on the workings of the mind. Much of psychological research attempts to infer from the observed behavior of the individual what the mental processes must have been. This attempt to make an objective science of psychology not only limits the kinds of data acceptable to the psychologist but must necessarily make for many inaccuracies in his inferences about the nature of the mental processes.
>
> The handicap placed on the work of the psychologist by the type of data he regards as useful and scientific is well demonstrated in the field of individual differences. Much of the work in this field is based on the use of various types of tests. In these tests the subject is presented with questions which he is to answer and problems which he is to solve. After some mental effort, the subject submits his answers or solutions for appraisal by the tester. In addition, the tester may secure observations of the behavior of the subject while taking the test. From these data the tester will attempt to draw

conclusions about the mental characteristics of the subject. What is missing is information on the process by which the problems are solved. The methods of attack, the steps in the thinking process, the kinds of considerations used to make one choice rather than another, and the feelings and attitudes of the subject are neglected or given very little attention. The products of thought --the answers to the questions or the solutions to the problems--plus the observations may give the tester a fleeting glimpse into the complex processes of thought involved, but usually this is incomplete and, almost of necessity, inaccurate.

If it were established that the evidence on overt behavior or the products of thought had a clear-cut and definite relationship to the mental processes which produced them, the psychologist would appear to have ample justification for limiting his data to such objective and easily observed phenomena. However, even the most simple exploratory studies on responses to test questions reveal that the same solution or product of thought may be arrived at by distinctly different processes of thought. (pp. 1-2)

...

The educator, as well as the psychologist, assumes a correspondence between mental products and processes. He usually assumes that the individual who has the largest number of correct responses on the final examination is the individual with the best or more desirable quality of mental processes. Even in classroom discussions, the teacher is generally more concerned about the accuracy of responses than about the methods by which the student arrived at his responses or solutions. In large part, this preoccupation with accuracy of responses is a consequence of the difficulties of getting information about the processes of thought, the limitations of time, the large size of classes, and the pressure for coverage of subject matter. In spite of this emphasis on the products of thought, educators usually agree that good habits (or processes) of thought are the important and significant outcomes of education. Also, they would probably agree that the particular solutions or answers given to schoolroom problems are of little consequence except insofar as they serve to indicate the quality of the student's thinking. (p. 3)

Bloom and Broder were clearly correct. But at that time--1950--in the United States, the kind of "process" study which they proposed (and carried out) was considered unthinkable if not unmentionable. It wasn't "objective," it didn't use statistics, it could not be of real value.

Some Americans, of course, persevered in pursuing independent lines of research (cf., e.g., the work of Easley, Erlwanger, and Ginsburg), but for several decades this line of attack was almost completely ignored. In part, there were almost no journals that would publish typical results, and probably none at all that could reach appropriate audiences. Somehow, in Switzerland, Piaget fared better. In the Soviet Union, the general method of Bloom and Broder has been perhaps the most trusted (and most often used) method of research.

The value of comparing U.S. and Soviet work in this area is entirely clear. Kilpatrick, Steffe, Wirszup, and others have already taken sizeable--and valuable--steps in this direction. But the end is nowhere in sight. Much more use of "interview methods" or "process studies" is called for.

3) However, as Herbert Simon has pointed out, these very valuable "naturalistic" observations *must be related to an appropriate theoretical "rhetoric" (or set of conceptualizations), so as to provide for deeper levels of analysis.*

Fortunately, considerable progress is being made in developing such conceptualizations, especially (within the U.S.) in the work of Papert, Minsky, Michie, Brown, Simon, Matz, Rissland, Winston, Abelson, Bobrow, Winograd, Larkin, Freiling, McDermott, Rieger, Schank, Norman and Rumelhart, Charniak, Goldstein, Sussman, and others.[3] A very useful reference on this work is Boden (1977); see also Davis and McKnight (1979) and Davis, Jockusch, and McKnight (1978). While this work is arising in many different contexts--computer interpretation of pictures, robotics, computer advice to physicians on medical diagnoses, and language comprehension, for example--it is coming to be recognized as a unified field of study, usually either under the name of "artificial intelligence" or "cognitive science." Its specific application to the analysis of mathematical thought and to mathematical problem solving is being carried out by Matz, Brown, Simon, Papert, and others.

The chapters by Rachlin and Kantowski report on some similar work in the Soviet Union, and Michie has prepared a volume (presently in press) that deals extensively with artificial intelligence work in the U.S.S.R. Soviet success in programming computers to play expert chess provides evidence of the high quality of Soviet work in artificial intelligence (cf. Levy, 1976, on KAISSA, a Soviet chess-playing computer program).

4) Soviet work on the neurological basis of learning difficulties is of outstanding merit. Specific application of such methods to learning difficulties in mathematics is being carried out in the U.S. by Denckla (1979) and Davidson (1979). There already seems to

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[3]Publications by some of these authors are cited on the list of references.

be effective communication between the U.S. and the U.S.S.R. on research of this type--but perhaps less-than-satisfactory communication between researchers in this area and the general world of mathematics education. In this case it appears that barriers between disciplines are more serious obstacles than national borders and language differences are.

### E. *Curriculum Improvement*

In the United States, beginning around 1957, there was a sizeable effort to improve school curricula in mathematics and science. The size of this program grew, until by 1968 it had reached the level of 20 million dollars per year (Fey, 1979, p. 493). According to Fey's summary of the three large surveys: "(The) survey data indicate that prior to 1976-77 about 30 percent of the sampled districts had used one or more of the federally sponsored innovative curricula in mathematics, but only 9 percent were still using those materials in 1976-77" (data from Weiss, 1978).

Beyond any doubt, there is little impact of the "new math" curricula still visible in U.S. schools today. Fey presents the following table:

TABLE 1

Course Enrollments in Secondary Mathematics for 1972-73 and 1976-77 (in Thousands)

| | 1972-73* | 1976-77** |
|---|---|---|
| General Mathematics 9-12 | 2417 | 2563 |
| Business Mathematics 9-12 | 392 | 609 |
| Elementary Algebra | 2052 | 2825 |
| Advanced Algebra | 1808 | 1317 |
| Geometry | 1506 | 1900 |
| Trigonometry | 171 | 460 |
| Probability/Statistics | 25 | 39 |
| Computer Mathematics | 63 | 153 |
| Advanced Senior Mathematics | 259 | 225 |
| Calculus | 55 | 105 |

*Source: Osterndorf, 1975
**Source: Weiss, 1978

This table has a number of interesting features, of which we call attention to just a few.

> Enrollment in business mathematics is up substantially. No NSF or other educational program has been active in this area. Fey attributes the increase at least partially to the increase in interest in consumer affairs, and partially to the increased desire to acquire job-relevant skills.

> The substantial increase in elementary algebra may be due to the increase in curricula that take two years to cover one year's work in order to accommodate less able (or less motivated) students; as a result, some students are in effect counted twice in computing "enrollment in elementary algebra." (Of course, a larger number of less-able students may actually be enrolled, in addition to the "double counting.")

> The respectable increase in calculus enrollments might be cited as further (small) evidence that students *can*, indeed, learn more mathematics than most schools teach.

> The small enrollments for statistics are interesting, since considerable emphasis was placed on statistics by some of the "new math" projects (though extensive teaching of statistics was ignored or opposed by some other "new math" projects).

> The notable increase in computer mathematics speaks volumes: once again, technology has a larger impact than rhetoric, logic, or persuasion.

> Nobody seems able to explain the very large increase in trigonometry enrollments (but with further data the mystery might be dispelled: for example, to what extent is it due to women realizing the importance of studying more mathematics?).

A number of worthwhile innovative curriculum improvements were developed in the United States. Their impact has been somewhere between slight and insignificant. Not only has this phenomenon not been explained, it has not really been studied, although the three studies summarized by Fey have made an important beginning.

Anyone familiar with the Madison Project in the United States must be struck by similarities between it and the Soviet curriculum revision effort described in Zankov's *Obuchenie i Razvitie* (Eksperimental' No-pedagogicheskoe Issledovanie) (1975), abridged and translated into English under the title *Teaching and Development.*

1) To begin with, the Madison Project started with trial teaching in a few classrooms; the teaching was done by the curriculum developers themselves. Indeed, the lessons were, in a sense, developed "right in the classroom with the students," primarily in Weston, Connecticut. Essentially the same approach was used in the

Zankov study, at School No. 172 in Moscow, with N. V. Kuznetsova as teacher. ("It would have been a mistake," Zankov writes, "to organize the experiment in a large number of classes at the same time. It was very important that, in the first stage..., each segment of the teaching process be thought out beforehand and later analyzed." The Madison Project view was identical. But perhaps especially significant is the fact that, *even in English*, one can read more about Zankov's program than about the Madison Project program.)

2) Trials were gradually extended, over a period of several years, to provide experience with more diverse student populations, and (more importantly) to allow for the training of a few more teachers who could become experienced with the new methods and materials. In Zankov's "second stage," in 1961, nine teachers were involved: S. M. Budyleva, K. D. Vasil'eva, L. S. Vinogradova, S. P. Mescheriakova, Z. E. Mitoropol'skaia, T. I. Orlova, Z. A. Snopkova, A. I. Sharapova, and M. M. Kosheleva. (Notice in what detail we can read about this Soviet "new curriculum" project!)

3) In subsequent expansion of the Madison Project program to more schools and more classes, the original teachers, by now quite experienced, formed "leadership cadres." They, and not books, were the main repository of the Project curriculum. As they trained more teachers, the "leadership cadres" expanded, ultimately numbering thousands of teachers, and extending from Connecticut to California. Various reports indicate that Soviet expansion also relied heavily upon experienced teachers.

4) The second major repository of the Madison Project program was a set of films, showing actual (and typical) classroom lessons, that could be analyzed and imitated by teachers who wished to learn the program. Zankov also used direct recording in several forms, including audio-tape recording and photography. ("We tried," Zankov writes, "to use photography to the broadest and fullest extent possible. Since photographs were taken by laboratory personnel, the camera lens captured the true life of the pupils, in all its naturalness and lack of constraint. Deliberate posing for the camera was excluded." Every word of that could have been written by Madison Project personnel, as well.)

5) In the United States, typical practice assigns teachers a relatively stereotyped role--for example, many textbooks are arranged so that whenever the book is opened, the two facing pages present *exactly* one lesson, never more, never less--but U.S. practice does not monitor how teachers deal with that role.

The Madison Project reversed this: the basic role assigned to teachers was a flexible one, often requiring diagnosis of individual student needs, and improvised planning in the classroom; but the Project attempted to provide quite detailed guidance on how this role could be handled. In this, the Project probably stood somewhere

between typical U.S. practice and typical Soviet practice (cf., e.g., the chapter by Romberg).

6) Both Zankov and the Madison Project extended grade levels upward quite gradually. Each project needed students who had completed the new curriculum for grade N before it was possible to *create* the curriculum for grade N + 1.

7) Related to that, both Zankov and the Madison Project conducted their basic R and D work *by following the same students for four or five years*, since both programs were concerned primarily with relatively long-term effects on students.

8) Both Zankov and the Madison Project report, on good evidence, that students were able to learn substantially more mathematics than had previously been thought possible (or had been attempted).

9) Indeed, even the mathematical content of the Madison Project curriculum resembles the present (1979) Soviet curriculum in many particulars.

10) Zankov reports that, as the implementation of his program matured, he found himself becoming more interested in the problems of implementation within the established social structure (and constraints) of the schools, and in the influence of home family life on students. Both statements are true also for the Madison Project (cf., e.g., Davis and Douglas, 1976).

### F. *Obstacles to Curriculum Improvement*

The problem of helping a student--or a class--to learn mathematics is, at least in general, a problem that can be solved, and has been, repeatedly. There is good evaluative data on the effectiveness of student learning (cf., e.g., Dilworth, 1973). The alleged "failure" of the "new mathematics"--*insofar as student performance is concerned*--is a myth. The best of the "new math" programs, when implemented faithfully, produced improvements *both* in student computational skill *and also* in student conceptual understanding (cf. Dilworth, 1973).

*The problem of teacher education was also solved*--at least in the sense that generally competent teachers who wanted to master the new programs, and who attended teacher education programs designed for this specific purpose, were in most cases able to use the new materials effectively (again, cf. Dilworth, 1973). Hence, at least in this *existence theorem sense*, two fundamental problems were solved: (1) creating curricula to improve student learning and (2) teaching teachers to use these curricula.

The problems which were NOT solved, and *still* have not been, in the U.S. (and presumably not in the U.S.S.R.), are the two problems Zankov refers to:

> How can one carry out an effective implementation of a new curriculum, within the constraints and confines of an on-going school program?

> How can one understand, and deal with, the very great effect that home environment and neighborhood peer influences have on student self-concept, aspirations, habits, and--ultimately--performance?

G. *The Educational Establishment and the Professional Teacher*

1. *Assumptions*

The difficulties of implementing new curricula represent a major problem. To leave the problem unsolved is to concede defeat, to allow the curricula to become increasingly ineffective and increasingly obsolete, increasingly unresponsive to the economic needs of society and to the personal needs of individuals. But solving the problem depends upon many things, among them the expectations we have for school systems, for universities, and for the role of individual teachers.

Here, clearly, different expectations are possible. Different expectations already exist, as Romberg points out, between U.S. and Soviet practice. But even more alternatives can be imagined, as Romberg again points out.

My personal preference is to look toward genuinely *professional* teachers, with an expanded role, an improved education, and a better work situation. Let me spell this out briefly, drawing on my personal experience:

Imagine a school where at least some of the teachers are *part-time* teachers. (This is not novel: we expect something of the sort from every major university, where a combination of scholarship, research, consulting, and teaching is the normal order of things.)

Imagine, perhaps, that it is a *small* school (size is a whole story in itself), with one or two senior mathematics teachers, assisted by several younger junior colleagues.

This organization matches my personal experience and my own personal needs. As a young teacher (at M.I.T.), I was expected to teach a prescribed curriculum; even the precise list of homework problems to assign each day was prescribed. *In this sense, it was a very Russian system.* But, as a novice, I probably found this reassuring and helpful. Today, as a mature teacher with over 30 years experience, I would find this too confining. Such external specification would deny me the right to use my own professional judgment, carefully developed by 30 years of studying, teaching, and experimenting.

Today I am inclined to say: I am a true professional, and I want to be treated as such. (Fortunately, I am. Today I can design the mathematics curriculum for my classes, and even to a considerable extent for my school, a high school that is officially a laboratory school.) A division of a small mathematics department into junior and senior staff, with curriculum design the prerogative of the senior staff (which is the arrangement in our laboratory school), is one possible arrangement.

Whatever assumptions you make about the nature of schools and the role of teachers, *you must make some*. Using *your* assumptions, consider some of the aspects of the implementation of new curricula, such as the following.

2. *Teacher Attitudes*

Teacher attitudes vary considerably, as anyone who has worked in implementation programs can testify. There are teachers who have independently arrived at programs very similar to the specific innovation in question, and who are delighted to find themselves suddenly in fashion. There are teachers who recognize many of the shortcomings of existing curricula, and who hunger for something new. And there are teachers who doubt the value of the innovation, or who believe that it challenges some of their cherished beliefs, or who just plain don't like it. Consider these statements from teachers, gathered by the surveys:

> I still think America came farther and faster than any nation in history under the old method of teaching, where we had some discipline in the classroom, we did some drill because it was what teachers deemed was necessary, we didn't have to try to justify all that we did. (Steffensen, 1978, pp. 33-34)

(Among several themes that I, personally, hear in this single statement is a plea from a serious professional teacher to be allowed to make some independent decisions, to be accepted as a qualified professional.)

> I am using the rote method pretty much because they have found out that with a three-level you can spend all hour trying to get them to understand (and they can't). (Denny, 1978, p. 47)

> I dislike our book, not enough drill, it's modern math. We adopted a new book...it has more drill, more basics and I'll like it. (ibid., p. 11)

> The study of mathematics takes levels of maturity. You have to take step one before you can take step two. There is a definite hierarchy of material. You can't hope to be creative until you've mastered the basic program of studies. (Mary Lee Smith, 1978, p. 104)

> I am a very traditional teacher. I use chalkboard, a textbook, and handouts. (Denny, 1978, p. 104)

Fey (1979) summarizes the attitudes of many teachers: "The current mood, as reflected in the NSF studies, is a search for stability, choosing from among topics and techniques with which teachers have long experience and comfortable familiarity" (p. 500).

Again, when I match this against my personal experience--not as a curriculum innovator, but as a *teacher*--I feel a considerable responsibility for my students, and I am reluctant to undertake *anything* that might prove harmful to them (including anything that might fail because I did not know how to make it work).

But one should bear in mind those teachers who have enthusiastically embraced the new curricula. Even thousands, among the more than a million U.S. teachers, don't figure prominently in random samples, but--since they were the teachers who *sought out* the innovative projects--they loomed *very* large in the experience of the various curriculum innovation projects.

### 3. *Institutional Constraints*

We mention only two, but there are many. A common experience of curriculum projects has been to find a (say) third-grade teacher who learned the project's materials, and used them well, only to pass on the students the following year to a fourth-grade teacher who "didn't believe in these new-fangled programs" and who refused to follow up on the new curriculum strands. In the U.S., this usually turned out to be an unsolvable problem. Strong curricula tend to require continuity, and continuity (in *new* programs) is hard to provide.

Where new curricula required new materials, such as MAB blocks or Cuisenaire rods, school ordering procedures usually implied *a delay of at least one year* before the materials could be available. And by then the teacher who wanted the material had frequently left that school, and the replacement teacher did not want the materials, nor know how to use them; the materials stayed in the closet.

### 4. *The Professional Teacher*

The autonomous professional teacher is not, in my view, an impossible myth. Consider these excerpts from statements by teachers:

> The NSF institutes that I attended were well worth all the money. I'm sure that if I had not attended these institutes I would not have been able to do as good a job as I have done...A college graduate with a degree in mathematics is not really prepared to teach high school. They don't

> teach you how to deal with kids and you also don't get much of a chance to get your head together with respect to math instruction. All math teachers should be encouraged to take an NSF institute at least every three to five years. (Serrano, 1978, p. 30)

> I can teach them to think logically about real problems in their lives today. (Denny, 1978, p. 111)

> Mathematics can teach the student how to think logically and that process can carry over to anything. To be able to start with a set of facts and reason through to a conclusion is a powerful skill to have. (ibid., p. 112)

Nearly all observers recognize the decisive role of teachers in our educational system:

> Any observer cannot fail to be impressed by the fact that the resources tied up in teaching here are almost entirely human resources. (Walker, 1978, p. 23)

When Louis Smith tried to elicit generalizations about education, he found that

> administrators, board members, teachers, pupils always ended up talking about individual teachers. (Louis Smith, 1978, p. 112)

And teachers very often *are* responsible professionals: Mary Lee Smith (1978) noted that one finds some teachers who have "kept open a window on the larger world of ideas" (p. 18). Robert Stake concluded that teachers

> saw themselves in a serious, not very exciting business; the business of education. They saw themselves as pretty good businessmen, wishing that times would change for the better, but confident that they could deliver on their promises and pretty well satisfied that there is not really a better way to run the shop. (Welch, 1978, p. 25)

5. *Teaching As An Art*

The United States has not been generally successful in studying and promoting the *art* of teaching. Yet in large part teaching *is* an art, and should be studied and perfected as an art. Fey (1979) comments:

> One very fundamental factor in the lives of teachers that seems to lead to their loss of enthusiasm and spirit of innovation is the isolation of a teacher's work in the classroom. In one sense this role as the sole adult in a sea of young people offers tremendous freedom and

> impressive responsibility. But on the other hand, it induces the separation of the world in school and the world outside of school that is seen by so many teachers and students to be a serious problem in education today." (p. 499)

To bring teachers into direct contact with the development of *teaching as an art* could be extremely valuable. Consequently, Kalmykova's study of *the art of teaching* as practiced by Petrova is a kind of R and D activity to which the U.S. should pay heed.

6. *Rewards and Motivation for Teachers*

A teacher's professional conscience can be a powerful motivating force. As far as improving the curricula is concerned, there is usually no other.

> Teachers tend to resist those things that make their job more difficult and are attracted to those things that make it easier or more effective. Resistance to innovation, appeal of teaching bright students, need for discipline, desire for smaller classes, resistance to administrative requests for personal goals are all explained by considering teaching in this light. (Welch, 1978, p. 13)

Presumably there are parallels with Soviet experience.

7. *Linear Constraints*

Can you change the curriculum in grade N without regard to the expectations in grade N+1? In many cases the answer is: no! Fey (1979) comments:

> A second, more subtle, factor inhibiting change is the reported prevalence at all levels of schooling of a "future orientation." Teachers feel a heavy responsibility to get students prepared for the next mathematics course. (p. 502)

8. *Summary: Curriculum Improvement*

It seems clear that curriculum improvement is a continuing and important need in education in the United States. It clearly is, also, in the Soviet Union, where reports indicate serious shortages of adequately trained personnel. The problem in the U.S. is far from solved, as Fey's excellent report makes painfully clear. One is led to assume that it must not be truly solved in the U.S.S.R.

*Here, then, is a common problem that both nations should be able to work on together. The similarity of many efforts thus far is most*

*striking! So, too (as Romberg's chapter makes clear), are the differences in underlying assumptions.*

Research and development workers in both nations are finding their interests turning, *out of necessity*, to the two "corollary" questions:

> How does one implement a curriculum innovation in mathematics?

> How does one learn more about, and cope better with, the influences of home environments and neighborhood peer groups?

For both of these questions, the differences between Soviet and U.S. assumptions are very great indeed! But that is precisely one reason why cooperation in studying the problems can be beneficial.

### *H. Forums for Serious Discussions*

In 1969, when Kilpatrick, Begle, Wirszup, Wilson, and their colleagues began the publishing, in English, of selected Soviet R and D reports, they made an important contribution to improved communication among researchers. Their work, fortunately, has continued, and should continue for the foreseeable future.

The matter should, however, go further. This is one part--though an important one--of a more general problem. Prior to 1969 there had been no satisfactory forum for the discussion of interview studies and process research: the prevalence of this approach within Soviet research was an eye-opener for many Americans. It helped to usher in a new era of U.S. research, as the perusal of recent NSF- and NIE-funded research projects makes clear. But how does the need for communication stand today? For brevity, we lump together seven communication needs:

a) The need for communication about *interview* and *process of thought* studies

b) The need for communication about *theoretical conceptualizations for the details of human information processing*, especially as applied to mathematical thought

c) The need for sharing *filmed, video-taped, and audio-taped records* of mathematical behavior

d) The need for forums for the discussion of the *philosophy of R and D work* in mathematics education

e) The need for *communication across national borders* (and, in the present case, between the U.S. and the U.S.S.R.)

f) The need for *R and D discussions that include teachers*

g) The need for broad and serious discussions of *curricula*, at every level from fine detail to broad outline and general goals

It would be quite beyond the scope of this paper to propose a solution to the over-all communication gap, but it is important to put U.S.-U.S.S.R. communication into the proper context. In 1969 the action of Kilpatrick, Begle, Wirszup, and Wilson was an ice-breaker: the communication gap was otherwise almost complete. In the decade since then, two relevant journals have appeared, *Cognitive Science* and *The Journal of Children's Mathematical Behavior*. In addition, two face-to-face forums were created to allow researchers to speak directly with one another: invitational sessions at the University of Pittsburgh and meetings of the International Group for the Psychology of Mathematics Education. Still further, special sessions at AERA meetings are coming to provide an important forum for discussions of research in the applications of artificial intelligence conceptualizations to the analysis of mathematical thought processes.

Beyond this, publications by research groups themselves have played an important role--for example, *A.I. Memos* from the Stanford University Computer Science Department and Technical Reports from the M.I.T. Artificial Intelligence Laboratory.

The communication situation might be summarized as follows:

| Need / Forum | Interview and Process Studies | Artificial Intelligence Applied to Math. Ed. | International | Films, Tapes | Philosophy, of R & D | Inclusion of Teachers | Curricula |
|---|---|---|---|---|---|---|---|
| Cognitive Science | | yes | | | | | |
| J. Child. Math Behav. | yes | yes | yes | | | ? | |
| Pittsburgh Sessions | yes | yes | somewhat | | some | | |
| IGPME | yes | yes | yes | yes | some | ? | |
| AERA | yes | yes | yes | | some | ? | ? |
| Russian translations | yes | ? | yes | | some | ? | ? |

Clearly, there are important communication gaps that still exist. In particular, major questions of curriculum choices have never been adequately discussed, and at present there is no clearly appropriate forum. The best answers may come from the National Council of Teachers of Mathematics. For more general involvement, one can *imagine* effective television discussion--the BBC has achieved this for various similar issues--but (despite attempts by Craig Fisher and others), effective television discussions of such matters seem not to exist in the U.S. *Broadening the communication problem to include the need to communicate between the U.S. and the U.S.S.R. can make the problem easier to solve, in part because Soviet alternatives can open up new and interesting possibilities.*

### I. *Final Summary*

We have adequate reason to study Soviet R and D work in mathematics education, in at least the areas mentioned in these chapters and probably others. Both the differences and the similarities can be of value. In developing programs of communication and cooperation, it is important to keep in mind the major problems that need to be solved in order to achieve more effective learning of mathematics, and to improve the lot of students, of teachers, and of graduates.

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Volume I: The Case Reports

Chapter 1: "River Acres" by Terry Denny
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Chapter 3: "Alte" by Louis Smith
Chapter 4: "BRT" by Alan Peshkin
Chapter 5: "Urbanville" by Wayne Welch
Chapter 6: "Pine City" by Rob Walker
Chapter 7: "Western City" by Rodolfo Serrano
Chapter 8: "Columbus" by James Sanders and Daniel Stufflebeam
Chapter 9: "Archipolis" by Jacquetta Hill-Burnett
Chapter 10: "Vortex" by Gordon Hoke
Chapter 11: "Greater Boston" by Rob Walker

Volume II: Design, Overview, and General Findings

Chapter 12: "The Various Aims of Science Education" by M. Steffensen
Chapter 13: "The K-12 Curriculum"
Chapter 14: "Pluralism and Uniformity"
Chapter 15: "Student Learning"
Chapter 16: "The Teacher in the Classroom"
Chapter 17: "The School and the Community"
Chapter 18: "Survey Findings and Corroborations" by Elizabeth Knight Dawson

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